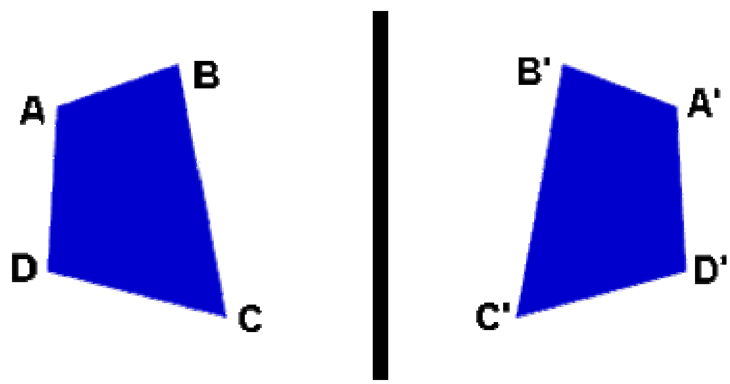
Transformation is a general term, which covers reflection, translation, rotation, dilation, line and point symmetries. We will look at them in more detail.

In mathematics, any transformation of an object is called its **image**. If the original object was labelled with letters, such as polygon ABCD, the image may be labelled with the same letters followed by a ***prime*** symbol, A'B'C'D'.

A **reflection** can be seen in water, in a mirror, in glass, or in a shiny surface. An object and its reflection have the **same shape and size**, but the **figures face in opposite directions**. In a mirror, for example, right and left are switched.



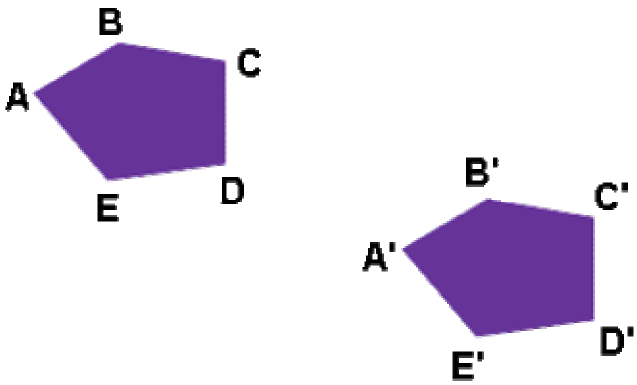
The line (where a mirror may be placed) is called the **line of reflection**. The distance from a point to the line of reflection is the same as the distance from the point's image to the line of reflection.

A reflection can be thought of as a "flipping" of an object over the line of reflection. To reflect an object means to produce its mirror image.

A **translation** "slides" an object a fixed distance in a given direction. The original object and its translation have the **same shape and size**, and they **face in the same direction**. The word "translate" in Latin means "carried across".

When you are sliding down a water slide, you are experiencing a translation. Your body is moving a given distance (the length of the slide) in a given direction. You do not change your size, shape or the direction in which you are facing. Translations can be seen in wallpaper designs, textile patterns, mosaics, and artwork.

To translate an object means to move it without rotating or reflecting it. Every translation has a direction and a distance.



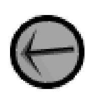
Think of polygon ABCDE as sliding two inches to the right and one inch down. Its new position is labelled A'B'C'D'E'.

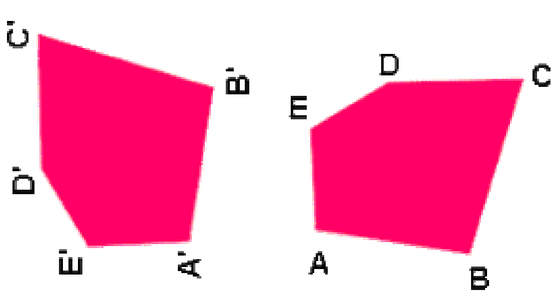
A translation moves an object without changing its size or shape and without turning it or flipping it.

A **rotation** is a transformation that turns a figure about a fixed point called the centre of rotation. An object and its rotation are the **same shape and size**, but the **figures may be turned in different directions.** Every rotation has a centre and an angle.

When you are riding on a ferries wheel, you are experiencing a rotation. Amusement park swings allow you to experience a rotation. Rotations can be seen in nature. The leaf on a plant illustrates the concept of a rotation. The centre of rotation is the point where the leaf is attached to the stem. Rotations can be seen in planetary movement

Rotations can occur in either a **clockwise** or **counterclockwise** direction.

Clockwise	Counterclockwise
	



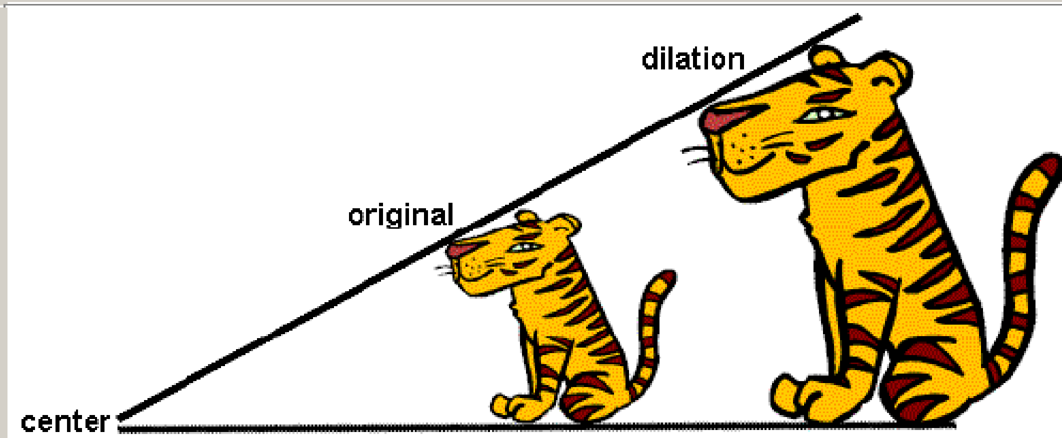
This rotation is 90˚ counterclockwise.

Notice that the *unit circle* moves in a **counterclockwise** direction. You will want to remember the layout of the unit circle when you are graphing figures and their rotations.

A **dilation** is a transformation that produces an image that is the **same shape** as the original, but is a **different size.**

A dilation used to create an image **larger** than the original is called an **enlargement**. A dilation used to create an image **smaller** than the original is called a **reduction**.

You are probably familiar with the word "**dilate**" as it relates to the eye. ***"The pupils of the eye were dilated."*** As light hits the eye, the pupil enlarges or contracts depending upon the amount of light. This concept of enlarging and contracting is "dilating".

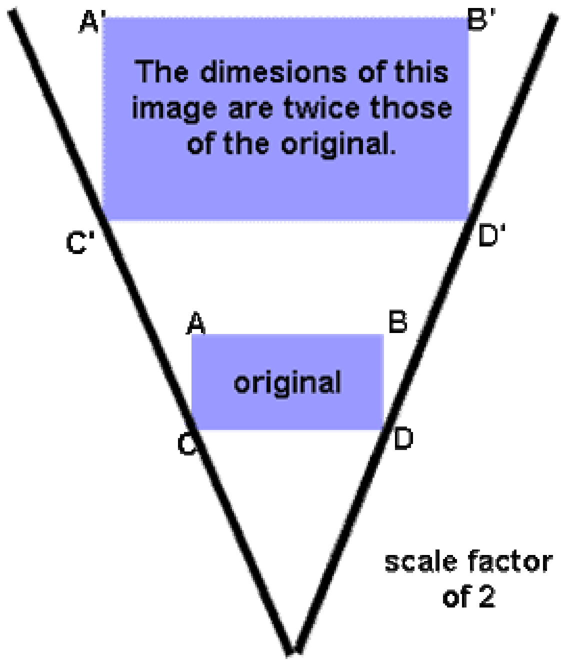


The description of dilation includes the **scale factor** and the **centre of the dilation**.

NOTE: the scale factor is sometimes referred to as the scalar factor.

If the scale factor h > 0, then the image X' of the point X lies on the half-line $\overrightarrow{PX}$, and |PX'| = h . |PX|

If the scale factor h < 0, then the image X' of the point X lies on the half-line $\overrightarrow{XP}$, and |PX'| = |h| . |PX|



If the scale factor is greater than 1, the image is an enlargement.

If the scale factor is between 0 and 1, the image is a reduction.

A figure and its dilation are **similar figures.**

The length of each side of the image is equal to the length of the corresponding side of the original figure multiplied by the scale factor. The distance from the centre of the dilation to each point of the image is equal to the distance from the centre of the dilation to each corresponding point of the original figure times the scale factor.

Remember: Dilations are enlargements (or reductions)

Line symmetry, or just **symmetry**, occurs when *two halves of a figure* mirror each other across a line. The **line of symmetry** is the line that divides the figure into two mirror images.

A simple test to determine if a figure has line symmetry is to fold the figure along the supposed line of symmetry and see if the two halves of the figure coincide.

Another name for the concept of line symmetry is **reflection**. Nature displays **line symmetry** in some of its most beautiful work. The balanced arrangement of symmetry creates pleasing and attractive forms. Many flowers possess **line symmetry.** The biologist's term for line symmetry is "bilateral symmetry." Certain letters of the alphabet and words possess line symmetry.

Notice that some possess vertical line symmetry, some possess horizontal line symmetry, and some possess BOTH vertical and horizontal line symmetry.

In mathematics, we often describe a concept like **line symmetry** with a formal definition. It may read something like the following:

Definition: A set of points has **line symmetry** if and only if there is a line, l, such that the reflection through l of each point of the set is also a point of the set.

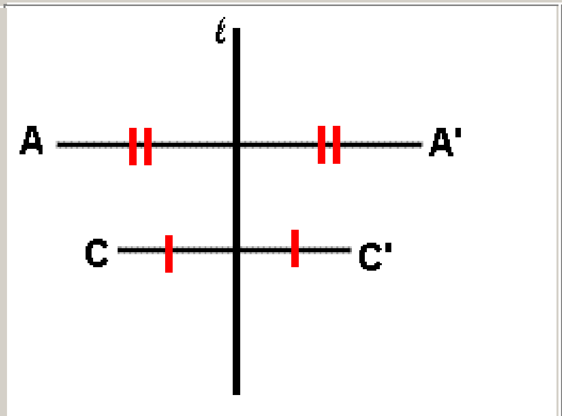
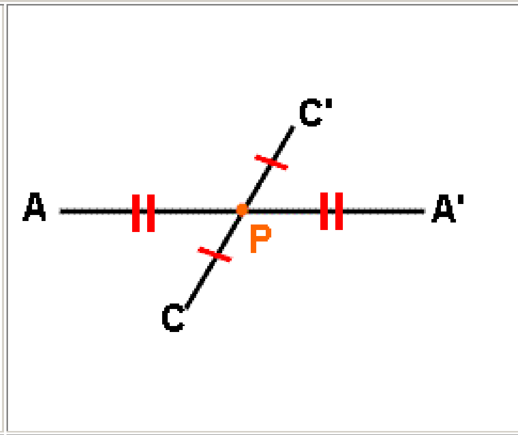
A figure has line symmetry if there is a line on which the figure may be folded so that the two parts of the figure will coincide.

An image of a point X in the line symmetry with the line o is a point X', for which it is valid that:

1. if $X \in o$, then $X \equiv X'$
2. if $X \notin o$, then $XX' \perp o$ and $|Xo| = |X'o|$

Point symmetry exists when a figure is built around a single point called the centre of the figure. For every point in the figure, there is another point found directly opposite it on the other side of the centre.

Study the diagrams below:

	
This figure shows two points and their reflections through a ***line***. This is line symmetry.	This figure shows two points and their reflections through a ***point.*** This is point symmetry.

In a point symmetry, the centre point is a **midpoint** to every segment formed by joining a point to its image. I.e. P ÷AA'. We say that A and A' are in a point symmetry with the point P.

A simple test to determine whether a figure has point symmetry is to turn it upside-down and see if it looks the same. A figure that has point symmetry is unchanged in appearance by a 180 degree rotation.

Geometric transformation/mappings in the plane

A transformation/mapping is every prescript, which to every point X in the plane assigns *exactly one* point X' from this plane. It does not matter, whether x is identical with X', or not. Point X is called the original figure. Point X' is its image.

A transformation is understood to be ***isometric***, if for two original figures X, Y and their images X', Y' it is true that: |XY| = |X'Y'|. I.e. Isometric mapping/transformation keeps distances.