

ALTERATED IMPLICATION always has the same truth value as the original implication, i.e.  $A \Rightarrow B = B' \Rightarrow A'$ , where  $B' \Rightarrow A'$  is the altered implication.

E.g. If the product of two numbers is even, then both numbers are not odd.

$$A \qquad \qquad \Rightarrow \qquad \qquad B$$

If two numbers are odd, then the product of these numbers is not even.

$$B' \qquad \qquad \Rightarrow \qquad \qquad A'$$

### QUANTIFIED STATEMENTS

Quantified statements are statements, which in connection with other expression or statements form a new statement, but with a different meaning. These statements contain so called *quantifiers*, which are of two types:

- **generality quantifier** -  $\forall$  for all ... it is true ...  
(It expresses the amount of elements of a given set, for which some property is common)
- **existential quantifier** -  $\exists$  there exists ..., for which is true ...  
-  $\exists!$  there exists *exactly one* ..., for which it is true  
(It expresses a guess of the amount of elements of the set, for which some property is common.)

Examples of statements containing quantifiers:

- There exists a trapezium, which is right-angled.
- $\exists x > 0; x + \frac{1}{x} = 2$
- $\exists n \in \mathbb{N}; n \leq 1$
- $\forall x \in \mathbb{R}; x^2 \geq 0$

### Negations of quantified statements

$$(\forall x; V(x))' \Leftrightarrow \exists x; V'(x)$$

$$(\exists x; V(x))' \Leftrightarrow \forall x; V'(x)$$

Example:

V: All multiples of 8 are even numbers.

V': There exists a multiple of 8, which is not an even number.