One characteristic of the mathematical language is its logic, exactness and explicitness.

A **mathematical statement** is every single utterance (grammatical sentence) about which it makes a sense to tell, whether it is *true* or *false*. That statement, which is true, is said to be valid, that one, which is false is said to be invalid. It is a message that is stated or declared; a communication (oral or written) setting forth particulars or facts, etc. Simply said, it is a statement which carries factual information, regardless of whether the information is correct or not. No statement may be both true and false at the same time in the same context.

The following utterances **are** statements:

1. Bratislava is the capital of Slovakia.
2. $2 + 5 = 6$
3. Chemistry deals with living organisms.
4. The sum of the length of two sides in a triangle is bigger than the length of the third side.

Statements 1. and 4. are true/valid, statements 2. and 3. are false/invalid.

The following utterances **are not** statements:

1. Good morning.
2. $x + 5 = 6$ (x is an integer)

Task: Decide on the following utterances – are they statements or not?

a) a student of a grammar school
b) the inner angles of an equilateral triangle have the size of 60°
c) $\Delta ABC \sim \Delta KLM$
d) $3 + 2\sqrt{2} > 0$
e) $5 + 2\pi - \sqrt{2}$
f) $a^2 + 2ab + b^2$
g) Every natural number is nonnegative.
h) $2x + y = 68$

Which of the math. statements are true?

Each statement you write should be logically and grammatically complete --- the main difference between mathematics and many other subjects is that we make use of many abbreviations such as ``$\Rightarrow$'' for ``implies'' and ``='' for ``equals'' or ``is equal to''. Using such abbreviations, you should be able to read aloud the work you have written in meaningful, complete sentences.

For example, if I simply write

$2x-4y = 6$,

$x-2y = 3$,

It is unclear whether I am considering a system of equations that I would like to solve, or I am claiming that the second equation is a logical consequence of the first. The first scenario can be expressed by writing ``Consider the system" before the equations. The second can be expressed by including the implication sign ``$\Rightarrow$" before the second equation.

The symbols ``=", ``$\Rightarrow$", and ``$\Leftrightarrow$":

You must use the symbols ``=", ``$\Rightarrow$", and ``$\Leftrightarrow$" correctly.

i. Equality: The ``=" symbol should be written between two mathematical objects that are the same (in calculus courses, these will be numbers or symbols representing numbers).

ii. Implication: The ``$\Rightarrow$" symbol is used to signify implication. It should appear between mathematical statements, not simply between numbers. Think of a mathematical statement as a complete phrase or something that can stand alone as a sentence, such as an equation. An example of implication is

$x = 2 \Rightarrow x^2 = 4$.

Notice that $x^2=4$ does NOT imply that $x=2$ (x might also be -2).

iii. Equivalence: The ``$\Leftrightarrow$" symbol is used to denote equivalence. It is used between two mathematical statements that imply each other and can be read as ``is equivalent to" or ``if and only if". For example,

$$\begin{aligned} & x^2 - x = 2 \\ \Leftrightarrow\ & x^2 - x - 2 = 0 \\ \Leftrightarrow\ & (x+1)(x-2) = 0 \\ \Leftrightarrow\ & x=-1 \quad \text{or} \quad x=2. \end{aligned}$$

NEGATION OF MATH. STATEMENTS

To each math. statement V it is possible to create a statement V’, which negates that meaning of the statement V. Such a statement V’ is called a **negation** of the statement V.

If V is true, then V’ is false. If V is false, then V’ is true.

Negation of a statement is a statement which has an opposite truth value to the original statement.

Statement	*Its negation*
Number 7 is divisible by 2.	Number 7 is not divisible by 2.
Today is Wednesday.	It is not true that today is Wednesday. Today is not W.

Certain phrases may cause problems, therefore we need to learn them:

statement	*negation of the statement*
every …… is……	at least one …… is not ……
at least one …. is …..	no one …. is …..
at least n ……. is ….	at most (n - 1) …. is ….
at most n ….. is …..	at least (n + 1) ….. is ….

<u>Examples</u>:

Every triangle is obtuse.	At least one triangle is not obtuse.
No abscissa is $\sqrt{2}$ cm long.	At least one abscissa is $\sqrt{2}$ cm long.
The equation 2x – 1 = 5 has at most one root.	The equation 2x– 1 =5 has at least 2 roots.

Operations with statements (compound statements)

1. conjunction (logical product) $A \wedge B$ …and, along with / simultaneously
2. disjunction/alternative (logical sum) $A \vee B$ … or …
3. implication/conditional $A \Rightarrow B$ implies, from that results/flows; if … then
4. equivalence/biconditional $A \Leftrightarrow B$ … iff… ; … right then …, if and only if

logical conjunctions

A	**B**	**$A \wedge B$**	**$A \vee B$**	**$A \Rightarrow B$**	**$A \Leftrightarrow B$**
1	1	1	1	1	1
1	0	0	1	0	0
0	1	0	1	1	0
0	0	0	0	1	1

Compound statement is a logical statement which is built from several simple statements.

E.g. 21 is divisible by 3 *and* 21 is not prime.

45 is a multiple *or9* or 13 – 20 = 7.

If 4 + 6 = 10 *and* 3 + 3 = 9, *then* all rectangles are squares.

When attempting to determine the truth value of a compound statement, first determine the truth value of each of the components it is composed of.

Mathematicians often use symbols and tables to represent concepts in logic. The use of these variables, symbols and tables creates a shorthand method for discussing logical sentences.

A truth table is a pictorial representation of all of the possible outcomes of the truth value of a compound sentence/statement. Capital letters such as A and B are used to represent facts (or statements) within the compound statement.

A **statement formula** is an expression which contains statements connected with conjunction, disjunction, implication and equivalence.

A **tautology** is a compound statement, which is always true, regardless the truth value of the statements it is composed of.

NEGATION OF COMPOUND STATEMENTS

COMPOUND STATEMENT	ITS NEGATION
$A \wedge B$	$A' \vee B'$
$A \vee B$	$A' \wedge B'$
$A \Rightarrow B$	$A \wedge B'$
$A \Leftrightarrow B$	$(A \wedge B') \vee (A' \wedge B)$

For more study materials, click the web side www.regentsprep.org, Math A, mathematical reasoning, Logic.