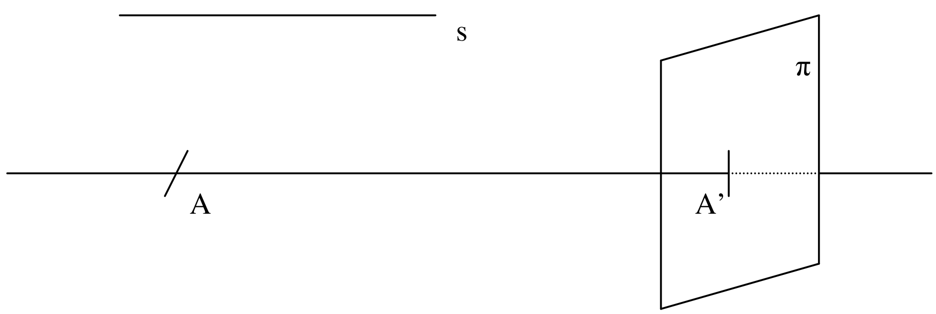
SOLID GEOMETRY

Parallel projection is given with the plane, usually marked with Greek letters α, β, γ, or π. It is also given with the direction of the projections s (of some line).

An image of a point in a parallel projection is found so that through the given point a line parallel with the direction of the projection is led. Its intersection with the plane π is the image of the given point in the parallel projection.



Properties of the PP:

1. An image of an abscissa in PP is:
   - a point – if the abscissa is parallel to the direction of PP
   - an abscissa
2. An image of two different parallel lines can be:
   - two different parallel lines
   - line
   - two different points
3. A ratio of the abscissas AB and CD equals the ration of their images
4. If the plane, which contains a pre-image/figure, is parallel to the plane π, then the given figure is identical with the image.

Basic terms from solid geometry

A *line* is unequivocally given by 2 different points.

A plane is unequivocally given by:

- three different point not lying on one line
- two parallel lines
- a line and a point that does not lie on this line
- two intersecting lines

Mutual position of two points A and B in 3D

- different, i.e. $A \neq B$
- coincident, i.e. $A \equiv B$

Mutual position of two lines in 3D

they **lie** in the same plane

they **don't lie** in the same plane

SKEW LINES

they **have** common points

they **don't have** a common point

PARALLEL (DIFFERENT) LINES

1 point

INTERSECTING LINES

number of points

COINCIDENT LINES

Mutual position of two planes in 3D

they intersect in a line

INTERSECTING PLANES

they do not intersect in a line

they intersect in a plane

COINCIDENT

they have no intersection

DIFFERENT

Mutual position of a line and a plane

they have a common point

they do not have a common point

PARALLEL

1 point

INTERSECTING

number of points

LINE LIES IN THE PLANE

Mutual position of three planes in 3D

1. Each two planes out of given three planes are parallel
2. Two planes are parallel, the third one intersects them in two parallel lines
3. Each two planes intersect in three parallel lines
4. Each two planes intersect in the same (common) line
5. Each two planes intersect in the same (common) point

Criteria on collinearity

1. There is one plane α only passing through a point parallel to the given plane β
2. There is one line p only passing through a point parallel to the given line q
3. If two lines a and b are both parallel to the same line p, then those two lines are parallel to each other
4. If two planes α and β are both parallel to the same plane γ, then those two planes are parallel to each other
5. If one of two parallel lines is parallel to a plane, then the other line is parallel to this plane as well
6. If a line is parallel to one of two parallel planes, then it is parallel to the other plane as well
7. If a plane contains two intersecting lines, which are parallel with another plane, then these two planes are parallel

SOLID SECTIONS

Section of a solid with a plane is an intersection of this solid with that plane. It is a plane shape and its border is an intersection of the solid edge with the section plane.

CUBE SECTION

Rule:

A plane intersects two other parallel planes in two parallel lines.

Example: Point P is an inner point of the edge BF in the cube ABCDEFGH. Construct the section of this cube with the plane EHP.

The plane EHP intersects the face ABFE in the abscissa EP. The planes ADE and BCG are parallel and the plane EHP intersects them, therefore according to the previous rule it intersects them in two parallel lines. The plane EHP intersects the plane ADE in the line EH, which means that the line of intersection of the planes BCG and EHP is a line passing through P and parallel to the line EH. Its intersection with the face BCGF is an abscissa PQ, which is at the same time an intersection of this face with the plane EHP. An intersection of the face CDHG with the plane EHP is obviously the abscissa HQ. The wanted section is the quadrilateral EPQH. Its sides EP and QH are parallel. Similarly we can prove that PQ and EH are parallel, therefore this quadrilateral is parallelogram as well.

SECTION OF A POLYHEDRON

When constructing polyhedron sections, we often utilize the following **rule**:

Let's have three planes, out of which each two intersect.

a) If two planes intersect in two lines of intersection, also the third one intersects them in one common point

b) If two lines of intersection are parallel, then the third line of intersection is parallel to them as well.

Example: Points P. Q, R are inner points of the edges HG, EH, BF of the cube ABCDEFGH. Construct the cube section with the plane PQR.

Plane PQR intersects the face EFGH in the abscissa PQ. What is the intersection of PQR with the face ABFE? One point of intersection is the point R, the other one will be constructed in accordance with the previous rule. Let's think about the planes EFG, ABE, PQR. Their lines of intersection are lines EF and PQ. Their point of intersection M lies on the line of intersection of the planes ABE and PQR. In this way the intersection of PQR and the face ABFE will be found out - it is an intersection of the line MR and square ABFE. Similarly intersection of PQR and BCGF will be constructed. The wanted section will be a pentagon PQURV. The lines UQ, RV and UR, PV are parallel.

PYRAMID SECTION

Pyramid is a solid, which has one significant face – its bottom base, and one significant vertex – the main vertex V. All the remaining faces of the pyramid are triangles, which are called side faces.

Bottom face and vertex V play an important role in pyramid sections. We usually construct the sections in the following way: We always consider three planes: section plane, bottom face and some side face. We start with that side face, where some point of the section plane lies. Similarly we proceed when constructing sections of a prism.

Example: In the plane of the bottom face of the pyramid ABCDV lies a line p and inside of the edge DV there is a point K. The line p is parallel to the edge AB and it is not parallel to any other bottom edge; moreover it doesn't intersect the bottom face. Construct the pyramid section with the plane pK.

Point K lies in the side face DCV. This plane intersects the bottom face in DC. This line of intersection has to have a common point with the line p, let's label it O. KO is the line of intersection of pK and DCV. We proceed in the same way with the side face BCV. Pairs of planes pK, ABC and BAV, ABC have parallel lines of intersection p and AB, therefore also the line of intersection of pK and BAV is parallel to p and AB.