Quadratic Inequalities

A quadratic inequality has the form:

$ax^2+bx+c \geq 0 \qquad ax^2+bx+c \leq 0$

$ax^2+bx+c > 0 \qquad ax^2+bx+c < 0$

Solve the quadratic inequality $x^2 - x - 12 < 0$

Solution

This inequality is solved with values of x that make x^2 - x - 12 negative (< 0).
The quantity x^2 - x - 12 changes from positive to negative or from negative to positive at the points where it equals 0. For this reason, first solve the equation

$$x^2 - x - 12 = 0$$

x^2 - x - 12 = 0 (x - 4)(x + 3) = 0 x = 4 or x = -3

Locating -3 and 4 on a number line, as shown in Figure 3, determines three intervals A, B, and C.
$(-\infty,-3)$, $(-3,4)$ and $(4,\infty)$

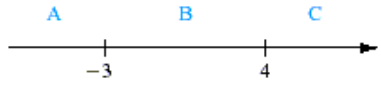


FIGURE 3

Decide which intervals include numbers that make expression **x^2 - x - 12** negative by substituting any number from each interval in the polynomial. For example,
choose -4 from interval A: $(-4)^2$ - (-4) - 12 = 8 > 0;
choose 0 from interval B: 0^2 - 0 - 12 = -12 < 0;
choose 5 from interval C: 5^2 - 5 - 12 = 8 > 0.
Only numbers in **interval B** satisfy the given inequality, so the solution is (-3, 4). A graph of this solution is shown in Figure 4.

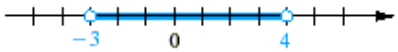
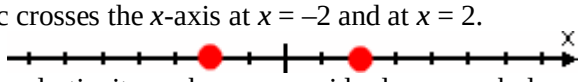
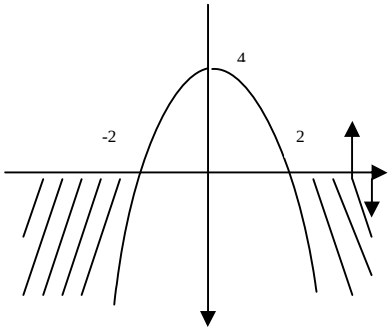


FIGURE 4

Solve $-x^2 + 4 < 0$.
I need to know where the graph crosses the *x*-axis. That is, I first need to find where $-x^2 + 4$ is equal to zero:
$-x^2 + 4 = 0$
$x^2 - 4 = 0$
$(x + 2)(x - 2) = 0$
$x = -2$ or $x = 2$
This says that the quadratic crosses the *x*-axis at $x = -2$ and at $x = 2$.



Since this is a "negative" quadratic, it graphs as an upside-down parabola.
a < 0



positive part of a plane

negative part of a plane

the result is an area enclosed by a curve and x-axis
intervals $(-\infty,-2)\cup(2,\infty)$

Quadratic inequalities Mgr. Oľga Štureková

Exercise

1, Solve each inequality:

1, $x^2 - 8x + 15 > 0$
2, $x^2 + 4x + 3 \leq 0$
3, $x^2 + x - 12 < 0$
4, $2x^2 + x - 3 < 0$
5, $6x^2 - 7x + 2 \geq 0$
6, $3 - 2x - x^2 < 0$
7, $x^2 - 2x - 8 > 0$
8, $x^2 - 2x - 15 \leq 0$

2, Solve each inequality:

1, $6x^2 - 7x + 2 \geq 0$
2, $x^2 + 4x + 3 \leq 0$
3, $x^2 + 4x + 3 \leq 0$
4, $x^2 + 3x \geq 10$
5, $x^2 + 21 < 10x$
6, $x^2 + 4 > 5x$
7, $54 - 11x \leq 30 - x^2$
8, $5x > 3 - x^2$

3, Solve each inequality:

1, $10x^2 + 40 \geq 0$
2, $2x^2 - 5x \leq 0$
3, $10 - 2x^2 \leq 0$
4, $x - x^2 \geq 0$

4, Solve each inequality:

1, $\frac{2x-3}{x^2-4} < 0$
2, $\frac{8x^2-48}{x+5} \leq 0$
3, $\frac{3x+7}{x^2-x-30} > 0$
4, $\frac{x^2-x-2}{x^2+4x} \geq 0$
5, $\frac{x^2-2x-8}{36-x^2} \leq 0$
6, $\frac{x^2-4x+4}{3x^2-5x+3} \leq 0$

5, Solve each inequality:

1, $\frac{x-3}{x+5} > \frac{x-5}{x+4}$
2, $\frac{2x+5}{2x-3} \leq \frac{4-x}{x+3}$
3, $\frac{x-11}{x+3} < \frac{x}{x-5}$
4, $\frac{x^2+9}{7-x} \leq \frac{x+4}{3}$