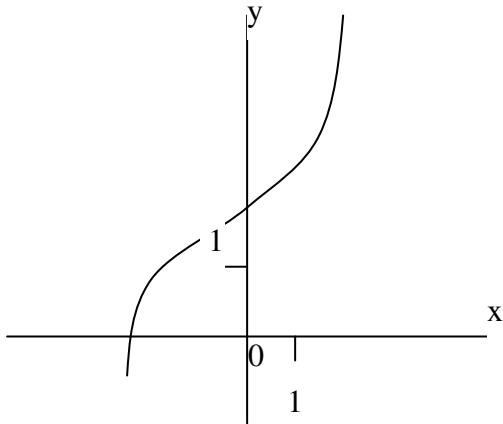
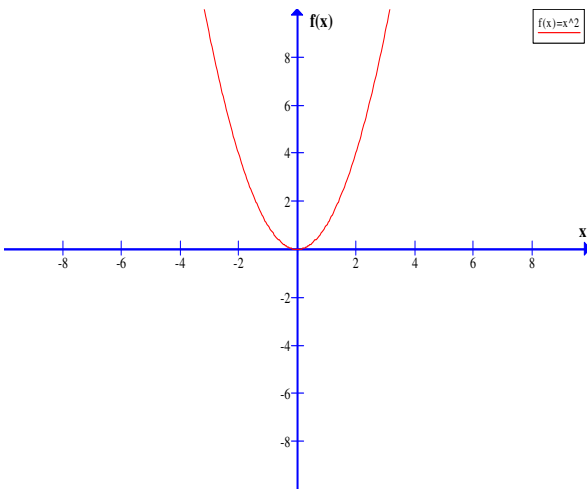


POWER FUNCTIONS

Power function is a function of the form: $f(x): y = x^n$, $n \in \mathbb{N}$, where x is the *base* raised to the *power* of n .

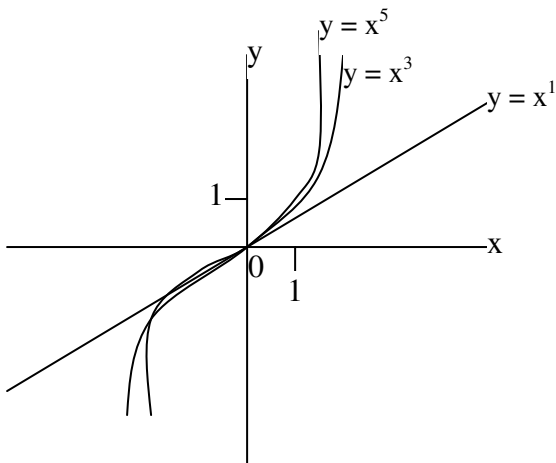
We distinguish positive and negative powers, moreover even and odd powers.

POWER FUNCTIONS WITH POSITIVE POWER

Power n is an odd number ($n = 1, 3, 5 \dots$)	Power n is an even number ($n = 2, 4, 6 \dots$)
The graph is a <i>cubic curve</i> :	The graph is a <i>parabola</i> :
	
Domain $D(f) = \mathbb{R}$, $D(f) = (-\infty, \infty)$	Domain $D(f) = \mathbb{R}$, $D(f) = (-\infty, \infty)$
Range of values $H(f) = \mathbb{R}$, $D(f) = (-\infty, \infty)$	Range of values $H(f) = \langle 0, \infty \rangle$
Increasing function on $D(f) = (-\infty, \infty)$	Increasing on $\langle 0, \infty \rangle$, decreasing on $(-\infty, 0)$
Odd function - for each x : $f(-x) = -f(x)$	Even function - for each x : $f(x) = f(-x)$
Not bounded function on $D(f) = (-\infty, \infty)$	Function bounded from below, not from above
no minimum, no maximum	Local minimum at point 0, no maximum

Ex. 1 Compare graphs of functions $y = x^1$, $y = x^3$, $y = x^5$.

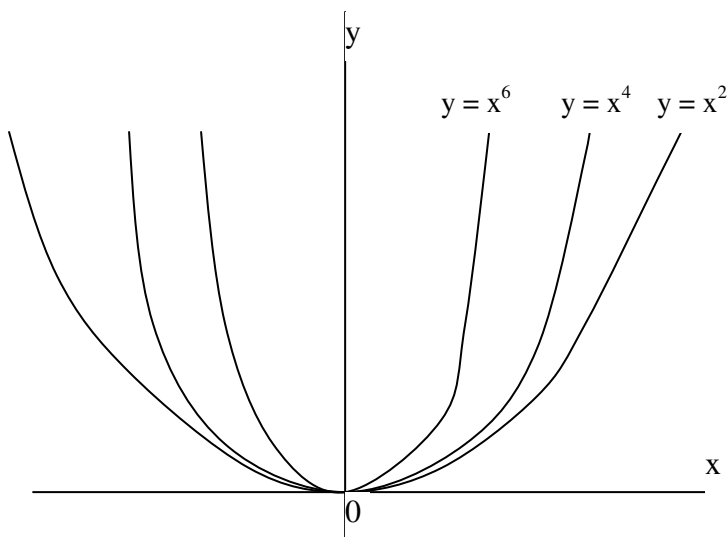
x	- 1.5	-1	- 0.5	0	0.5	1
$y = x^1$	- 1.5	-1	- 0.5	0	0.5	1
$y = x^3$	- 3.375	-1	- 0.125	0	0.125	1
$y = x^5$	- 7.594	-1	- 0.031	0	0.031	1



The power defines the slope of the graph. The higher power, the graph is nearer to y-axis.

Ex. 2 Compare graphs of functions $y = x^2$, $y = x^4$, $y = x^6$.

x	- 1.5	-1	- 0.5	0	0.5	1
$y = x^2$	2.25	1	0.25	0	0.25	1
$y = x^4$	5.063	1	0.063	0	0.063	1
$y = x^6$	11.391	1	0.016	0	0.016	1



REVISION :

Basic rules for calculating with powers:

$$x^n \times x^m = x^{n+m}$$

$$\frac{x^n}{x^m} = x^{n-m}$$

$$(x^m)^n = x^{m \times n}$$

$$(x_1 \times x_2)^n = x_1^n \times x_2^n$$

$$x^{-n} = \frac{1}{x^n} \quad x \neq 0$$

$$x^{\frac{n}{m}} = \sqrt[m]{x^n} \quad x \geq 0$$

POWER FUNCTIONS WITH *NEGATIVE* POWER

Formula: $y = x^{-n} = \frac{1}{x^n}$, $n \in \mathbb{N}$

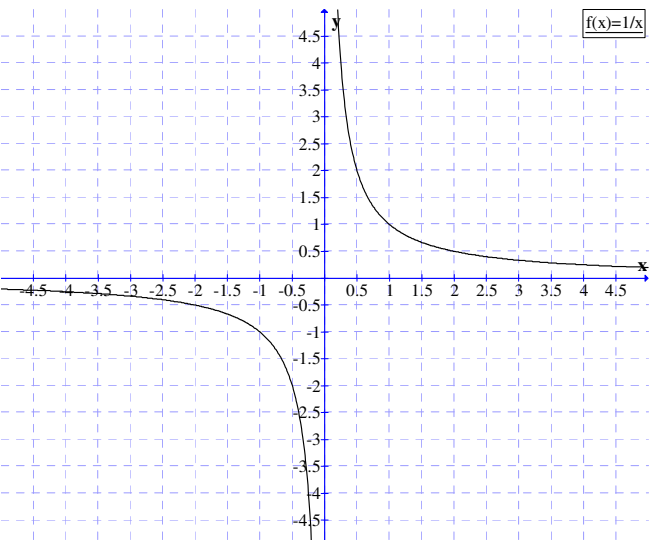
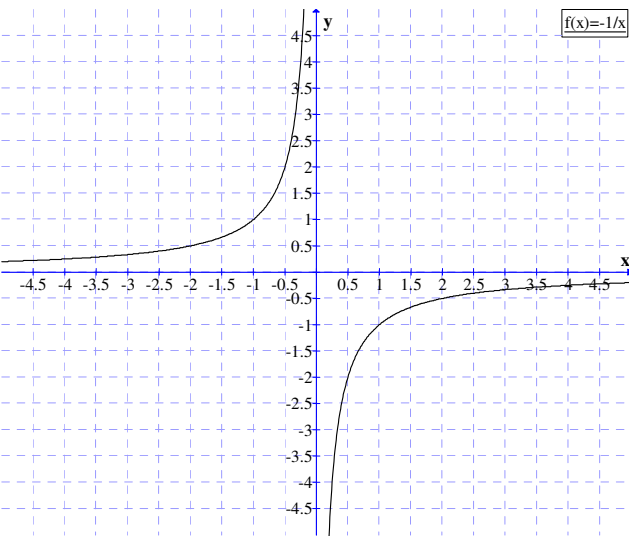
When the power is negative we apply the formula for calculation with a negative index, i.e. we will obtain a fraction, where in the denominator there will be the unknown x raised to positive power. Since the unknown x occurs now in the denominator, we need to consider the domain of the function – we know there cannot be zero in the denominator, i.e. the whole expression occurring in the denominator has to have its value different from 0.

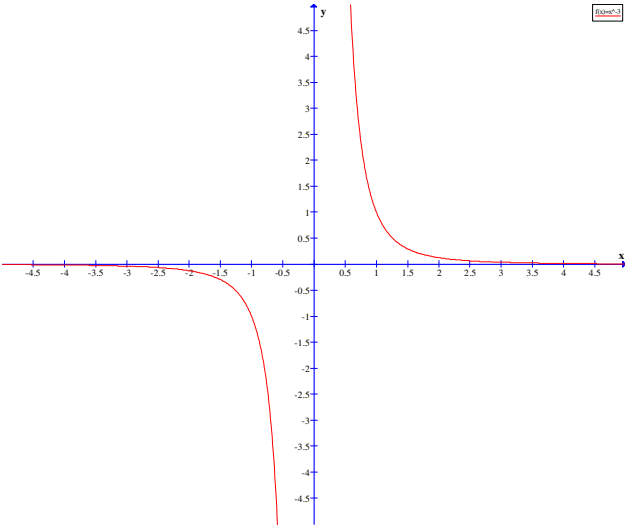
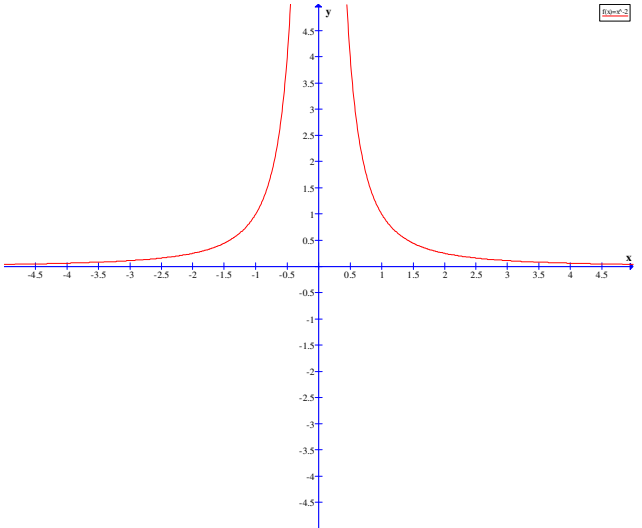
When the power is -1 the function is called the indirect ratio/proportionality, i.e. $x^{-1} = \frac{1}{x}$.

INDIRECT PROPORTIONALITY

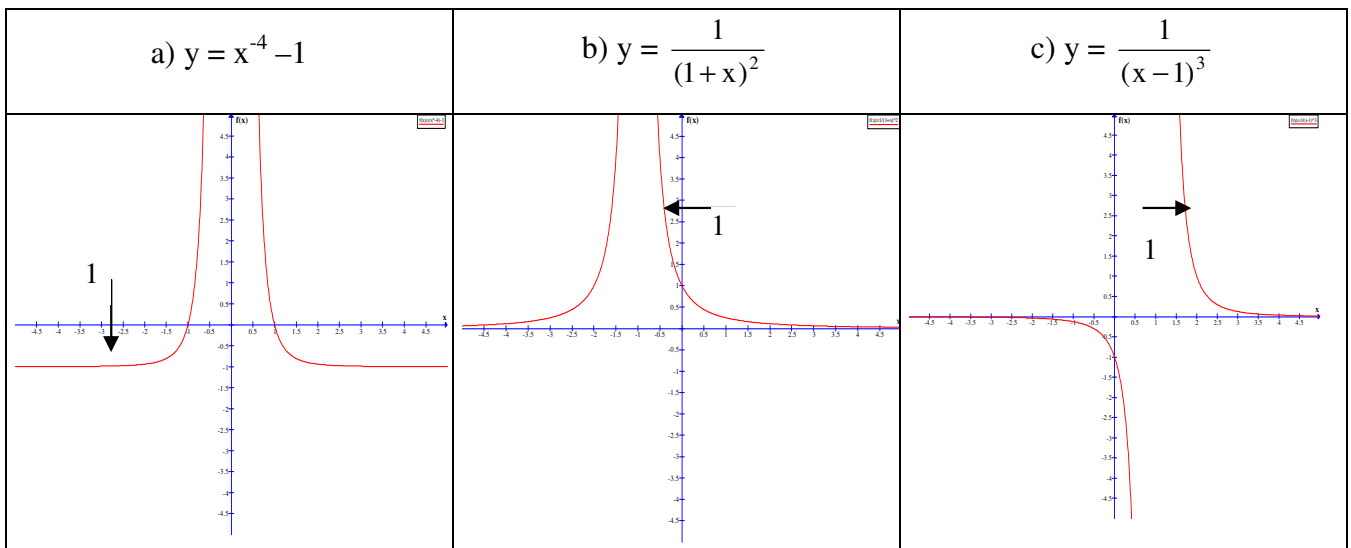
Definition: Let k be a real number, $k \neq 0$. **Indirect ratio** is the function given by formula $y = \frac{k}{x}$.

$D(f) = \mathbb{R} - \{0\}$. A graph of this function is a rectangular *hyperbola*.

$k > 0$	$k < 0$
Graph: 	Graph: 
Domain $D(f) = \mathbb{R} - \{0\}$, $D(f) = (-\infty, 0) \cup (0, \infty)$	Domain $D(f) = \mathbb{R} - \{0\}$, $D(f) = (-\infty, 0) \cup (0, \infty)$
Range of values $H(f) = \mathbb{R} - \{0\}$, $D(f) = (-\infty, 0) \cup (0, \infty)$	Range of values $H(f) = \mathbb{R} - \{0\}$, $D(f) = (-\infty, 0) \cup (0, \infty)$
Odd function – for each x : $f(-x) = -f(x)$	Odd function – for each x : $f(-x) = -f(x)$
Decreasing on $D(f) = (-\infty, 0) \cup (0, \infty)$	Increasing on $D(f) = (-\infty, 0) \cup (0, \infty)$
Not bounded function on $D(f) = (-\infty, 0) \cup (0, \infty)$	Not bounded function on $D(f) = (-\infty, 0) \cup (0, \infty)$
No minimum, no maximum	No minimum, no maximum

Power $-n$ is an odd number $n = -1, -3, -5\dots$	Power $-n$ is an even number $n = -2, -4, -6\dots$
The graph is a <i>hyperbole</i> :	Graph:
	
Domain: $D(f) = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, \infty)$	Domain: $D(f) = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, \infty)$
Range of values: $H(f) = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, \infty)$	Range of values: $H(f) = \mathbb{R}^+ = (0, \infty)$
Decreasing on $(-\infty, 0) \cup (0, \infty)$	Increasing on $(-\infty, 0)$, decreasing on $(0, \infty)$
Odd: for each x it's true: $f(-x) = -f(x)$	Even: for each x it's true: $f(-x) = f(x)$
Not bounded	Bounded from below with 0
no maximum, no minimum	no maximum, no minimum

Ex. 1 Draw graphs of functions a) $y = x^{-4} - 1$, b) $y = \frac{1}{(1+x)^2}$, c) $y = \frac{1}{(x-1)^3}$



LINEAR FRACTIONAL FUNCTIONS

Linear fractional function is every function of the form: $f(x): y = \frac{ax + b}{cx + d}$, where $a, b, c, d \in \mathbb{R}; c \neq 0$;

$ad - bc \neq 0$.

To sketch the graph of this kind of a function it is necessary to learn the division of terms because we shall divide the numerator with the denominator. Moreover we cannot forget to set the domain,

i.e. $D(f) = \mathbb{R} - \{-\frac{d}{c}\}$.

Division of terms:

$$(4x^3 + 5x^2 - 3x + 8) : (2x + 3) = 2x^2 - 0.5x - 0.75 + \frac{8.25}{2x + 3}$$

$$\underline{-(4x^3 + 6x^2)}$$

$$(-x^2 - 3x + 8)$$

$$\underline{-(-x^2 - 1.5x)}$$

$$(-1.5x + 8)$$

$$\underline{-(-1.5x - 2.25)}$$

$$8.25$$

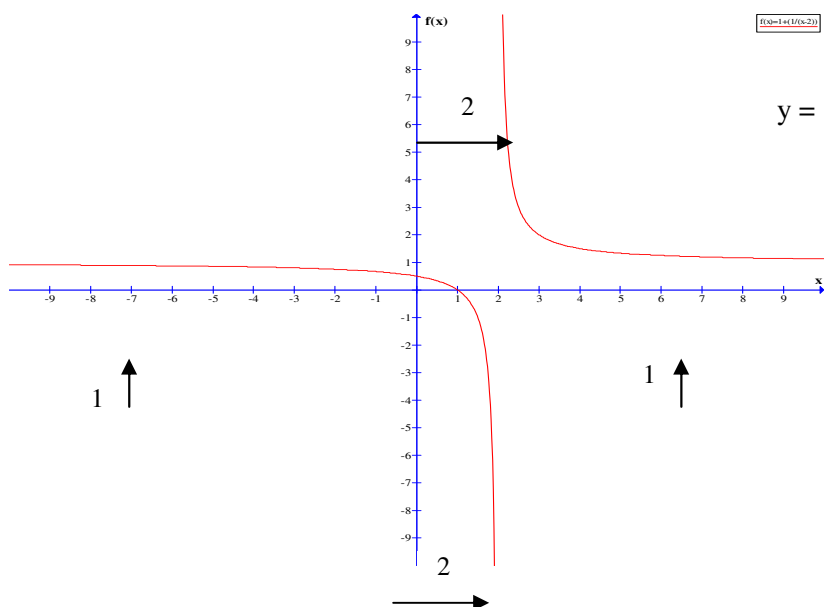
Ex. 1 Draw a graph of the function $y = \frac{x-1}{x-2}$

$$D(f) = \mathbb{R} - \{2\}$$

$$(x - 1) : (x - 2) = 1 + \frac{1}{x - 2}$$

$$\underline{-(x - 2)}$$

$$1$$



$$y = \pm a + \frac{k}{x \pm b}$$

+a	-a	+b	-b
↑	↓	←	→

A graph of linear fractional function is a hyperbola which is got by shifting the graph of function

$y = \frac{k}{x}$ (indirect proportionality).

The number **a** out of the fraction tells us the direction along y-axis, considering its sign, the number **b**, which is out of the domain due to the denominator, tells us the direction of the shift to the right or to the left along x-axis.