

Percentages

Percentages – Multipliers

The **multiplier** for an **increase of 6% is 1.06** ($100\% + 6\% = 106\%$ or 1.06).

To increase an amount by 6%, multiply it by 1.06.

The **multiplier** for a **decrease of 12% is 0.88** ($100\% - 12\% = 88\%$ or 0.88).

To decrease an amount by 12%, multiply it by 0.88.

For repeated percentage changes, multiply all the multipliers in turn.

Example:

A fast food restaurant decided to decrease by 18% the weight of packaging for their regular meals, which weighed 40 grams. Calculate the weight of the new packaging.

Method 1: Non-calculator paper

Find 18% of 40g: 10% of 40g = 4g
 1% of 40g = 0.4g So 8% of 40g = 3.2g
 So 18% of 40g = 4 + 3.2 = 7.2g

So new packaging would weigh $40 - 7.2 = 32.8\text{g}$

Method 2: Calculator paper (Multipliers)

To decrease by 18%, the multiplier is 0.82 ($100\% - 18\% = 82\%$).

So new weight is $0.82 \times 40 = 32.8\text{g}$

Example 2:

A manufacturer buys a machine for £56800. The machine is expected to **depreciate** by 12% in the first year and by 8.5% each future year. What will be the expected value of the machine after 5 years to the nearest £1?

Note: Depreciate means to lose value.

The multiplier for a decrease of 12% is 0.88.

The multiplier for a decrease of 8.5% is 0.915 (as $100\% - 8.5\% = 91.5\%$ or 0.915).

So value of machine after 5 years is

$$56800 \times 0.88 \times \cancel{0.915} \times \cancel{0.915} \times \cancel{0.915} \times \cancel{0.915} \times \cancel{0.915} = 56800 \times 0.88 \times 0.915^4 = \text{£}35036 \text{ (to nearest £1)}$$

multipliers for 2nd, 3rd, 4th and 5th years

Examination Question

A shop is having a sale. Each day, prices are reduced by 20% of the price on the previous day.

Before the start of the sale, the price of a television is £450.

On the first day of the sale, the price is reduced by 20%.

Work out the price of the television on

- the first day of the sale
- the third day of the sale.

Examination Question:

A car was bought for £7600. It depreciated in value by 25% each year. What was the value of the car after 3 years?

Percentage Change

You need to know the following formulae

$$\text{Percentage increase (or decrease)} = \frac{\text{Increase (or decrease)}}{\text{Original amount}} \times 100$$

$$\text{Percentage profit (or loss)} = \frac{\text{Profit (or loss)}}{\text{Cost price}} \times 100$$

Example:

In 1990 a charity sold $2\frac{1}{4}$ million lottery tickets at 25p each. 80% of the money was kept by the charity.

a) Calculate the amount of money kept by the charity.

In 1991, the price of a lottery ticket fell by 20%.

Sales of lottery tickets increased by 20%.

80% of the money obtained was kept by the charity.

b) Calculate the percentage change in the amount of money kept by the charity.

a) Money made by selling tickets = $2\frac{1}{4}$ million \times 25p = £562500
Money kept by charity = 80% of £562500 = $0.8 \times$ £562500 = £450000.

b) In 1991 price of ticket = 80% of old price = 80% of 25p = 20p
In 1991, sales of lottery tickets = 120% of 1990 sales
$$= \frac{120}{100} \times 2250000 = 2700000$$

In 1991, money made by selling tickets = $2700000 \times$ 20p = £540000
So amount to charity is $0.8 \times$ £540000 = £432000

Therefore:

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{Original amount}} \times 100 = \frac{450000 - 432000}{450000} \times 100 = 4\%$$

Examination Style Question:

Del 'boy' Rotter buys an old cottage for £84000. He spends £10400 on repairs and renovation then sells the cottage for £149000. Find his percentage profit to the nearest 1%.

Percentages- Compound Interest

Compound interest is interest paid on an amount and on the interest already received on that amount.

You can solve compound interest questions using the formula:

$$A = P \left(1 + \frac{R}{100} \right)^n$$

where: P is the amount invested initially;
 R is the rate of interest (percentage per year)
 n is the number of years invested;
 A is the amount in the account at the end.

Example:

A building society pays compound interest at a fixed rate of 7% per annum. If £480 is invested in an account, what will be the value of the account after 3 years?

Method 1:

In first year: Interest paid is 7% of £480 = $0.07 \times 480 = £33.60$
So balance at end of 1st year is $£480 + £33.60 = £513.60$.

In second year: Interest paid is 7% of £513.60 = $0.07 \times 513.60 = £35.952$
So balance at end of 2nd year is $£513.60 + £35.952 = £549.552$

In third year: Interest paid is 7% of £549.552 = $£38.46864$
So balance at end of 3rd year is $£549.552 + £38.46864 = £588.02$ (to nearest 1p)

Method 2: Use of formula

After 3 years balance would be $A = P \left(1 + \frac{R}{100} \right)^n = 480 \left(1 + \frac{7}{100} \right)^3 = 480 \times 1.07^3 = £588.02$.

Note: Sometimes you are asked for the total amount of interest that has been received. You get this by subtracting the initial amount invested from the final balance. Here the total interest received is $£588.02 - 480 = £108.02$.

Example:

Tony invests £500 in a bank account that pays 4% (compound) interest p.a. By what single number must £500 be multiplied by to get the amount in the account after 5 years?

The amount invested after 5 years is $P \left(1 + \frac{R}{100} \right)^n = 500 \times \left(1 + \frac{4}{100} \right)^5$.

So the required multiplier is $(1.04)^5 = 1.21665$ (to 5 decimal places).

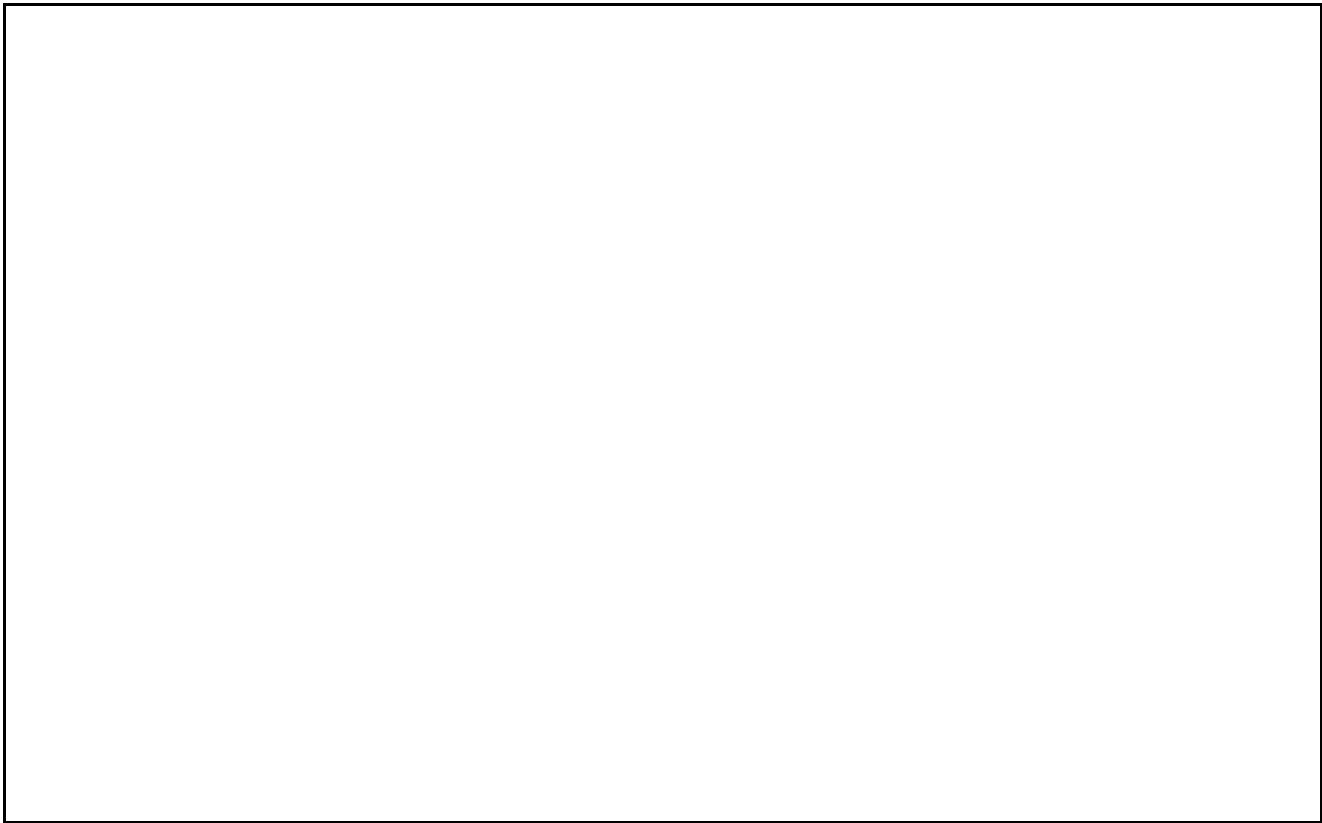
Examination Question:

£500 is invested for 2 years at 6% per annum compound interest.

a Work out the total interest earned over the two years.

£250 is invested for three years at 7% per annum compound interest.

b By what single number must £250 be multiplied to obtain the total amount at the end of the 3 years?



Examination Question:

Nesta invests £508 in a bank account paying compound interest at a rate of 10% per annum. Calculate the total amount in Nesta's bank account after 2 years.

Reverse Percentage Questions

Calculating the **original value** of something **before an increase or decrease took place** is called “calculating a reverse percentage”.

Example 1:

Ella bought a pair of climbing boots for £45.60 in a sale that gave “20% off”. What was the non-sale price of the boots.

Let the original price of the boots be 100%. The sale price then is 80%.

So 80% of the price = £45.60

So 1% of the price = $£45.60 \div 80 = £0.57$

So 100% of the price = $£0.57 \times 100 = £57$.

So the non-sale price of the boots was £57.

Examination Question 1:

A clothes shop has a sale. All the original prices are reduced by 24% to give the sale price.

The sale price of a jacket is £36.86.

Work out the original price of the jacket.

Examination Question 2:

In a sale all prices are reduced by 16%. Alan buys a shaver in the sale for £21.

How much does he save by buying it in the sale?

[Hint: First work out the price before the sale]

Example 2:

The total price of a bike (including VAT at 17.5%) is £146.85.

Calculate the cost of the bike excluding VAT.

Let the cost of the bike before VAT be 100%. Then the cost including VAT would be 117.5%.

So 117.5% = £146.85.

Then 1% = $£146.85 \div 117.5 = £1.249787\dots$

So 100% = $£1.249787 \times 100 = £124.98$ (to nearest 1p).

So the cost of the bike excluding VAT is £124.98.

Examination Question 3:

The price of a new television is £423. This price includes Value Added Tax (VAT) at 17.5%.

a) Work out the cost of the television **before** VAT was added.

By the end of each year, the value of a television has fallen by 12% of its value at the start of that year. The value of the television was £423 at the start of the first year.

b) Work out the value of the television at the end of the **third** year.

Examination Question 4:

The population of a town increased by 20% between 1981 and 1991. The population in 1991 was 43200.

What was the population in 1981?

[Hint: Let the population in 1981 be 100%].

Examination Question 5:

A tourist buys a stereo which costs £155.10, including VAT at 17.5%.

Tourists do not have to pay VAT. How much does the tourist pay?