PARAMETRIC EQUATIONS

If we have equations such as:

2(x-1) = 1(x-1)

2(x-3) = 9(x-1)

2(x-2) = 4(x-1)

2(x-5) = 25(x-1)

we can notate them briefly in a form:

$2(x-p) = p^2(x-1)$ where $p \in \{1,2,3,5\}$

This equation is called a **parametric equation** in the unknown x and a **parameter p.**

The parametric equation is the notation of all equations, which we gain by putting the constants {in our case 1,2,3,5} for the parameter p from its definition set.

If we extend the definition set of parameter into the whole R, we gain a set with the indefinite number of equations notated with the one parametric equation.

From the choice of parameter depends not only the solution of the equation, but also the number of solutions.

To solve the parametric equation we need to assign a set of roots to each value of parameter. We try to make x stand alone. Always check the solutions! In the end we have to discuss the solutions considering the parameter.

Ex: $\frac{2x-a}{x-5} = a$ / (x – 5) D = R – {5}

2x – a = ax – 5a

2x – ax = - 4a

x (2 – a) = - 4a / (-1)

x (a – 2) = 4a We think of two possible cases:

$a = 2$	$a \neq 2$
$x \cdot 0 = 8$	$x = \frac{4a}{a-2}$
$0 \neq 8$	$S = \{ \frac{4a}{a-2} \}$
$S = \{ \}$	

QUADRATIC EQUATIONS WITH A PARAMETER

From the choice of a parameter depends whether we get a linear or a quadratic equation. Therefore we need to take into account both possibilities. E.g.:

$ax^2 + (2a + 3)x + a + \frac{3}{4} = 0$

a = 0

linear equation

$3x + \frac{3}{4} = 0$

$x = -\frac{1}{4}$

$a \neq 0$

quadratic equation with a discriminant:

$D = (2a + 3)^2 - 4a(a + \frac{3}{4}) = 9a + 9 = 9(a + 1)$

D > 0

a > -1

$x_{1,2} = \frac{-2a - 3 \pm 3\sqrt{a+1}}{2a}$

D = 0

a = -1

$x = \{\frac{1}{2}\}$

D < 0

a < -1

x = { }

INEQUALITIES WITH A PARAMETER

We solve these inequalities similarly as we solve equations. Here however we need to pay attention to the sign of inequality – it changes when we multiply or divide by a negative number!

Exercise: $p - 2x \geq 2px - 1$

$p + 1 \geq 2px + px$

$p + 1 \geq 2x\,(p + 1)$

p = -1

$0 \geq 2x.0$

$0 \geq 0$

S = R

$p \neq -1$

p < -1

$1 \leq 2x$

$\frac{1}{2} \leq x$

$x \in \left\langle \frac{1}{2}, \infty \right)$

p > -1

$1 \geq 2x$

$\frac{1}{2} \geq x$

$x \in \left(-\infty, \frac{1}{2} \right\rangle$

EQUATIONS WITH TWO PARAMETERS

We solve the equations at the beginning for one parameter, and then we state conditions for the second parameter as well.