NUMERALS

- Arabic: 0,1,2,3,4,…
- Roman: I, II. III. IV, X, L, C, D, M, …

Numerical systems

- decimal: 0,1,2,3,4,5,6,7,8,9
- binary: 0,1
- ternary: 0,1,2

constant = invariable: a letter which stands for one number only, e.g. $\pi = 3.14$

variable: a letter which stands for a set of numbers, usually it's a letter **x, y, y,** etc.

Exercise:

a) sum of numbers x, y multiplied with 5 (x+y) 5
b) deduct/subtract 8 from the product of m, n m.n - 8
c) divide number u with the sum of numbers v, 6 u / (v+6)
d) add the division of numbers a and b to their double sum a/b + 2ab
e) add triple product of any number to its power $3x + x^2$
f) triple the difference of second powers of two real numbers $3 (x^2- y^2)$
g) Make the second power of 7-multiple of sum of two real numbers $[7(x+y)]^2$
h) Think of a number, add 5 to it, then the result multiply with 2 and deduct/subtract 10. (x + 5)2 - 10

NUMERICAL SETS

Natural numbers N include 1 and all its successors : 1,2,3,4,5,…

Natural numbers and zero N_0 include 0 and N

Whole numbers Z are the natural numbers, zero, and the reverse numbers to the natural ones: -2, -1, 0, 1, 2, …They are also called integers.

Rational numbers Q are all numbers which can be written in the form of a fraction: $\frac{10}{2}$

Irrational numbers I cannot be written in the form of a fraction, e.g. π , $\sqrt{2}$

Real numbers R are Q $\cup$ I

Complex numbers C are real numbers R together with the imaginary number **i =** $\sqrt{-1}$

Write the number 1234 in the decimal numerical system: 1*1000 + 2*100 + 3*10 + 4*1

What is the cipher sum of 1287? 1+2+8+7= 18

DIVISIBILITY OF THE NATURAL NUMBERS

DEF: a natural number **a** is a multiple of a natural number **b** even when exists such a natural number **k** that **a = k*b,** i.e. **b/a**.

Sentence: Each natural number n can be written by means of a natural number b > 1 in one form only as n = k*b + z, where $k \in N_0$ (partial division)

and $0 \leq z < b$ (remainder)

A natural number is divisible with	10	when it ends with the cipher 0
	5	0 or 5
	2	0,2,4,6,8 (even No)

A natural number is divisible with 4 (20, 25, 50) when its last two numbers are divisible with 4 (20, 25, 50)

A natural number is divisible with 3 (9) when its cipher sum is divisible with 3 (9).

A natural number is divisible with	6	when it is divisible with 2 and 3 simultaneously.
	12	3 and 4 simultaneously.

Rules:

1/n, n/n, n/0,

$a/b \wedge b/c \Rightarrow a/c$

PRIME NUMBER is each natural number, which has exactly two different factors only: 1 and itself.

COMPLEX NUMBER is each natural number which has at least three different divisors.

REDUCTION (DECOMPOSITION) OF A COMPLEX No means to express the complex number by means of the product of its divisors bigger than 1. A number may have various decompositions. E.g.

60 = 2*30= 4*15 = 5*12 = 2*3*10 = 2*2*3*5.

PRIME NUMBER DECOMPOSITION of a complex number is a notation of the complex No in the form of a product whose each factor is a prime number.

Prime numbers: 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47, …

The factors of a number that are prime are called **prime factors**.

A multiplication of prime factors is called a **product of primes** > we break a number down into factor pairs until a product of primes is reached.

- THE COMMON MULTIPLES of 2 numbers are those that are multiples of both, e.g.

multiples of 4 are 4,8,12,16,20,24, …

multiples of 6 are 2, 12,18,24,30,36, …

Common multiples of 4 and 6 are 12,24,36, …

THE LEAST COMMON MULTIPLE OF 4 and 6 is 12.

- THE COMMON FACTORS of 2 numbers are those that are factors of both, .e.g.

factors of 4 are 1,2, and 4

factors of 6 are 1,2, 3, and 6.

Common factors are 4 and 6 are 1 and 2.

THE HIGHEST COMMON FACTOR of 4 and 6 is 2.

Prime products can be used to find highest common factors and least common multiples, e.g. of No 72 and 60.

72 = 2*2*2*3*3

60 = 2*2*3*5

2 appears at least twice and 3 appears at least once in each prime product, so the *highest common factor* is 2*2*3 = 12.

2 appears at most three times in a prime product, 3 appears at most twice in a prime product, and 5 appears at most once in a prime product, so the *least common multiple* is 2*2*2*3*3*5.

DECIMALS

If we express numbers as decimals, a rational number will either have a finite number of decimal places or will recur. An irrational number will have an infinite number of decimal places without recurring.

In recurring decimal notation, dots are placed over the first and last of the set of recurring digits. E.g. 0.236666666… = 0.236

Any fraction is equivalent to a terminating or recurring decimal. E.g. $\frac{11}{30}$ = 11/30 = 0.3666…=0.36

APPROXIMATION: numbers can be approximated to a set number of decimal places:

32.37<u>4</u>3 to 2 decimal places (2dp) is 32.37

34.49<u>6</u> to 2dp is 34.50.

Rounding to a set number of significant figures is another form of approximating: 32.3743 to 2 significant figures (2sf) is 32.

346 534.7945 to 3sf is 347 000, to 7sf 346 534.8.

FRACTIONS

Two fractions equal in value are called **equivalent** fractions. $\frac{1}{2}$ and $\frac{5}{10}$

A fraction in its **lowest terms** is an equivalent fraction where the numerator and denominator have no common factors except 1.

An **improper fraction** is one where the numerator is larger than the denominator, e.g. $\frac{11}{4}$. An improper fraction is greater than 1, so it can be written as a **mixed** number, a mixture of whole numbers and fractions. $\frac{11}{4} = \frac{8}{4} + \frac{3}{4} = 2\frac{3}{4}$

- To find equivalent algebraic fractions, multiply or divide the numerator (top) and denominator (bottom) by the same number or expression. E.g.

$\frac{n}{3n} = \frac{1}{3}$; $\frac{n}{3n} = \frac{3n}{9n}$

- To add or subtract algebraic fractions with different denominators:
  - find a common multiple of the denominators
  - find equivalent fractions with a common multiple as the new denominator
  - add or subtract the fractions
- To multiply algebraic fractions, multiply the numerators and multiply the denominators.
- Two numbers that multiply together to give 1 are **reciprocals** of each other (if we have 2, its reciprocal is $\frac{1}{2}$)
- To divide algebraic fractions, use the rule that dividing by a fraction has the same effect as multiplying by its reciprocal.

To write a fraction as a decimal, divide the numerator by the denominator:

$\frac{3}{8}$ = 3/8 = 0.375

To write a decimal as a percentage, multiply the number by 100: 0.375*100 = 37.5 %.

PERCENTAGES

To calculate a percentage of a given amount, multiply the amount by the appropriate decimal.

E.g. To calculate 65% of 420 kg: 420 * 0.65 = 273. So 65% of 420 kg is 273 kg.

Writing one number as a percentage of another: compare the numbers 18 and 36:

18 is half of 36 and 18/36 = 0.5

18 is 50% of 36 or 0.5 * 100=50%

As a single calculation (18/36)*100 = 50%.

ALGEBRAIC EXPRESSIONS

For all real number *a, b* it is true that:

$a^2 + 2ab + b^2 = (a+b)\ (a+b) = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)\ (a-b) = (a-b)^2$

$a^3 + 3a^2b + 3ab^2+b^3 = (a+b)\ (a+b) = (a+b)^3$

$a^3 - 3a^2b + 3ab^2-b^3 = (a-b)\ (a-b)\ (a-b) = (a-b)^3$

$a^2 - b^2 = (a+b)\ (a-b)$

$a^3 + b^3 = (a+b)\ (a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)\ (a^2 + ab + b^2)$

Rational pointed expression is a quotient of 2 polynomials, where the denominator is different form the zero polynomial. We always have to set the solution set.

MULTIPLYING OUT BRACKETS – we need to multiply every term inside the bracket by the term outside, e.g.

$2(3a^2 + 4a – 6) = 2*3\ a^2 + 2*4a –2*6 = 6\ a^2 +8a – 12$ These are equivalents expressions, since the equality is true for any value of a.

You can **multiply terms** by grouping **numbers**, and each of the **letters**, e.g. 2m*3n = 6mn
To **simplify an expression** collect together any like terms, e.g. $2a^2b + 3ab^2 + 4a^2b = 6a^2b + 3ab^2$

You **factorise an expression** by looking for common factor of the terms and write the expression using brackets, e.g. $3a^2 + 6ab = 3(a^2 + 2ab) = 3a(a + 2b)$ This is factorised fully as there are no other common factors.
We factorise a numerical factor, an algebraic factor, or both.

To **factorise** an expression is to write it as a multiplication of its factors.
Rule: quadratic expression $ax^2 + bx + c = a * (x - x_1) * (x - x_2)$

It is true that $x_1 * x_2 = \frac{c}{a}$

$$x_1 + x_2 = \frac{-b}{a}$$

By adding more monomials we reach polynomial, i.e. $t^3 – 2t + 7$ is a sum of t^3 , – 2t, 7.
In the monomial –2y the numeral –2 is a *coefficient* and y is a *variable*.

CHANGING THE SUBJECT OF FORMULAS

Let's have a formula that links distance **s** with time **t**: $\mathbf{s = \frac{1}{2} g t^2}$ > s is the subject of this formula. To calculate t for the value of s, we need to make t the subject of the formula.

$s = \frac{1}{2} g t^2$ s is the subject /*2

$2s = g t^2$ /g

$\frac{2}{g}s = t^2$ / $\sqrt{\ }$

$\sqrt{\frac{2}{g}s} = t$ t is the subject of formula