

MATRICES

A matrix stores mathematical information in a concise way. The information is written down in a rectangular array of rows and columns of terms, called **elements** or **entries**, each of which has its own precise position in the array.

Notation: We normally represent matrices by capital letters. For example A, B, M, etc.

The order of a matrix is its shape.

- ⇒ When stating the order of a matrix, we must always give first the number of rows, followed by the number of columns.
- ⇒ $\begin{pmatrix} 4 \\ 8 \\ 7 \end{pmatrix}$ is a **column** matrix and has order 3 x 1, since its elements are arranged in three rows and only one column.
- ⇒ the matrix (4 8 7) has order 1 x 3 and is a **row** matrix.
- ⇒ when the number of rows and the number of the columns are equal, the matrix is called a square matrix, i.e. it has order n x n.

Addition and Subtraction of Matrices

We can add or subtract two matrices only when they are of the *same order*. We add or subtract an element from matrix A with that element from matrix B, which is on the same position.

Multiplication of matrices

- ⇒ To multiply a matrix by any real number **k** we multiply every element of the matrix by **k**.
- ⇒ To multiply a matrix by another one we have to take into account the following rule: The number of columns in the first matrix must be the same as the number of rows in the second matrix.

To multiply **A** by **B**, we start by taking the first row of matrix **A** (2 3 1) and the first

column of matrix **B** $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

We then multiply the first element of the row by the 1st element of the column, the 2nd element of the row by the 2nd element of the column, and the 3rd element of the row by

the last element of the column. We then add up these three products. This gives the element in the top left-hand corner of the matrix \mathbf{AB} , which is $2.1 + 3.1 + 1.0 = 5$

Next we take the first row of matrix \mathbf{A} and the second column of matrix \mathbf{B} , which gives us the 2nd element of the first row of matrix \mathbf{AB} .

Generally, the product \mathbf{AB} produces a matrix which has the same number of rows as \mathbf{A} , and the same numbers of columns as \mathbf{B} . Hence, if \mathbf{A} has order $a \times t$ and \mathbf{B} has order $t \times b$, then \mathbf{AB} has order $a \times b$.

- ⇒ Multiplication of two matrices is **not commutative**. That is, $\mathbf{AB} \neq \mathbf{BA}$. Therefore we must ensure that we write matrices in the correct sequence.
- ⇒ Multiplication of any three matrices \mathbf{A} , \mathbf{B} and \mathbf{C} is associative, i.e. $\mathbf{A(BC)} = (\mathbf{AB})\mathbf{C}$