LINEAR FUNCTIONS

Definition: **Linear function** is each function given by formula f: $y = mx + c$; m, c $\in$ R; D(f) = R

If x = 0, then y = c, i.e. c is the functional value of the function at point 0 (f(0) = c)

Linear function given by formula $y = c$ is a **constant function**. A graph of this function is a straight-line parallel to x-axis, cutting the y-axis at point [0, c].

It's valid: $m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$, where m is the gradient = slope of the line which shows us, whether

1. the function/line is increasing $\Leftrightarrow$ m > 0
2. the function/line is decreasing $\Leftrightarrow$ m < 0
3. the function/line is constant $\Leftrightarrow$ m = 0

Ex: Find all linear functions that contains the ordered pairs [0,2] and [-3,5]

Formula of linear function is $y = mx + c$, where m, c are unknown constants now. We have to substitute into the formula for x and y. Then we have to solve the simultaneous system of equations.

$y = mx + c$

2 = m . 0 + c $\Rightarrow$ c = 2

<u>5 = m . (-3) + c</u>

5 = -3m + 2

m = -1

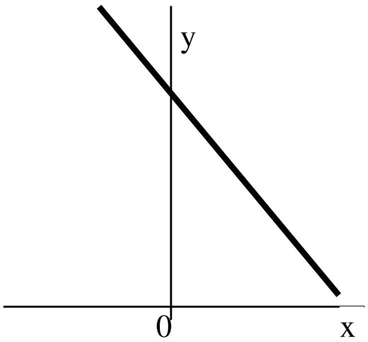
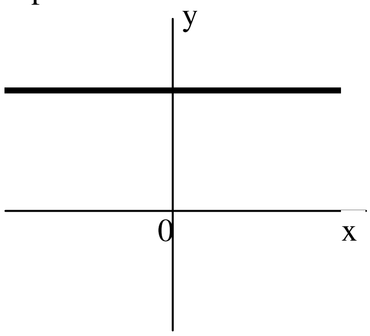
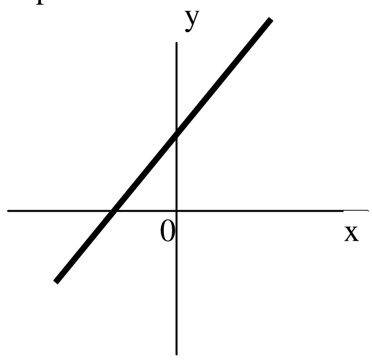
f: y = -x + 2

Other way of solving this is to calculate gradient m firstly according to the formula:

$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{5-2}{-3-0} = -1$

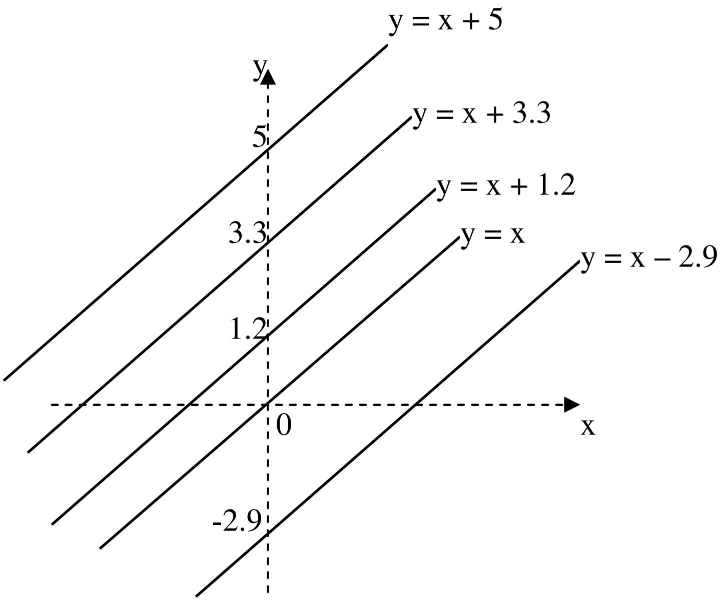
So the linear function is y = -x + 2 because for x = 0 is y = -0 + 2 = 2.

PROPERTIES OF LINEAR FUNCTION f: y = mx + c

If m < 0	**If m = 0**	**If m > 0**
Graph: 	Graph: 	Graph: 
Domain: D(f) = R	Domain: D(f) = R	Domain: D(f) = R
Range of values: H(f) = R	Range of values: H(f) = {b}	Range of values: H(f) = R
Decreasing function – for each x: If $x_1 < x_2$ so $f(x_1) > f(x_2)$	Not increasing, not decreasing – for each x: If $x_1 < x_2$ so $f(x_1) = f(x_2) = c$	Increasing function – for each x: If $x_1 < x_2$ so $f(x_1) < f(x_2)$
One-to-one function – for each x: If $x_1 \neq x_2$ so $f(x_1) \neq f(x_2)$	Many-to-one function – for each x: If $x_1 \neq x_2$ so $f(x_1) = f(x_2) = c$	One-to-one function – for each x: If $x_1 \neq x_2$ so $f(x_1) \neq f(x_2)$
Unbounded function	Bounded function	Unbounded function
Function has neither maximum nor minimum	Function has a maximum and minimum at each point $x \in D(f)$	Function has neither maximum nor minimum

Ex: We have a set S of all linear functions according to y = x + c if $c \in R$. Draw graphs of few of these functions.

Graphs of functions from set S are straight-lines parallel to straight-line defined by function y = x and c is a parameter that defines distance from the origin of coordinate system.



Ex: We have a set S of all linear functions according to y = mx + 3 if m∈ R. Draw graphs of few of these functions.

Graphs of functions from set S are straight-lines that pass through the point [0,3]. Parameter m defines the slope of straight-lines.