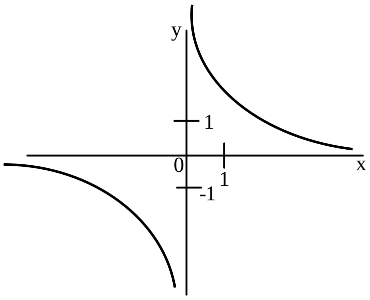
Linear fractional function

$$y=\frac{P(x)}{Q(x)}=\frac{ax+b}{cx+d}=\frac{k}{x};(k\neq 0)$$

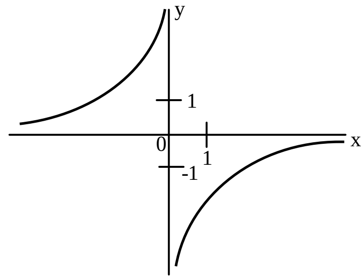
P(x) and Q(x) are polynomials and $Q(x)\neq 0$, $a,b,c,d\in R;c,d\neq 0$ and $ad\neq bc$;(if $ad\neq bc$ then it is constant function)

- The graph of this function is **hyperbola**

k > 0



k < 0



Domain = R- {0}
Range = R- {0}

Decreasing on the intervals $(-\infty,0),(0,\infty)$

Increasing on the intervals $(-\infty,0),(0,\infty)$

Not bounded above neither below
No maximum, no minimum
Odd

Example

Draw the graph of the function $y=\frac{3-x}{x-1}$ and write the properties.

Divide (*3-x)* by (*x* – 1)

Look just at the leading *x* of the divisor and the leading -x of the dividend.

$(-x+3)\div(x-1)=-1$

Now I'll take that *x*, and multiply it through the divisor, *x* - 1.

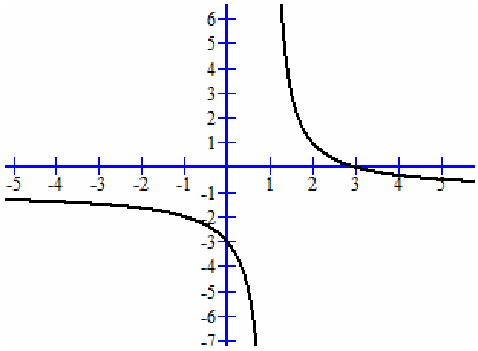
$$\underline{(-x+3)}\div(x-1)=-1$$
$$-x+1$$

To <u>subtract</u> the polynomials, **I *change all the signs* in the second line**

$$\underline{(-x+3)}\div(x-1)=-1$$
$$\underline{-(-x+1)}$$
$$0+2$$

So I can write $y = \frac{3-x}{x-1} = -1 + \frac{2}{x-1}$

D(f) = R - {1}; H(f) = R - {-1}, decreasing, not even, not odd, one-to-one, not bounded, no Min, no Max



Exercise

Draw the graphs of the following functions and write their properties

a) y = $\frac{2x+3}{x-1}$,

b) y = $\frac{-x+2}{x-1}$,

c) y = $\frac{2x-4}{-x+2}$

d) y = $\frac{x+3}{x-1}$,

e) y = $\frac{5-2x}{3x-1}$

f) y = $\frac{3x+3}{2x-4}$