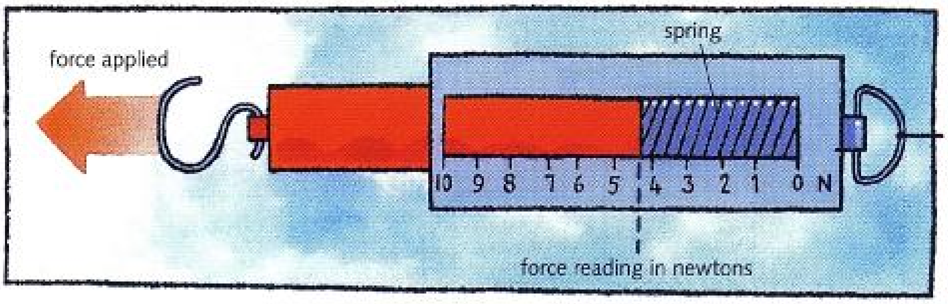
2.07 Forces in balance

Typical forces in newtons

force to switch on a bathroom light......	10 N
force to pull open a drinks can......	20 N
force to lift a heavy suitcase.....	200 N
force from a large jet engine....	250 000 N

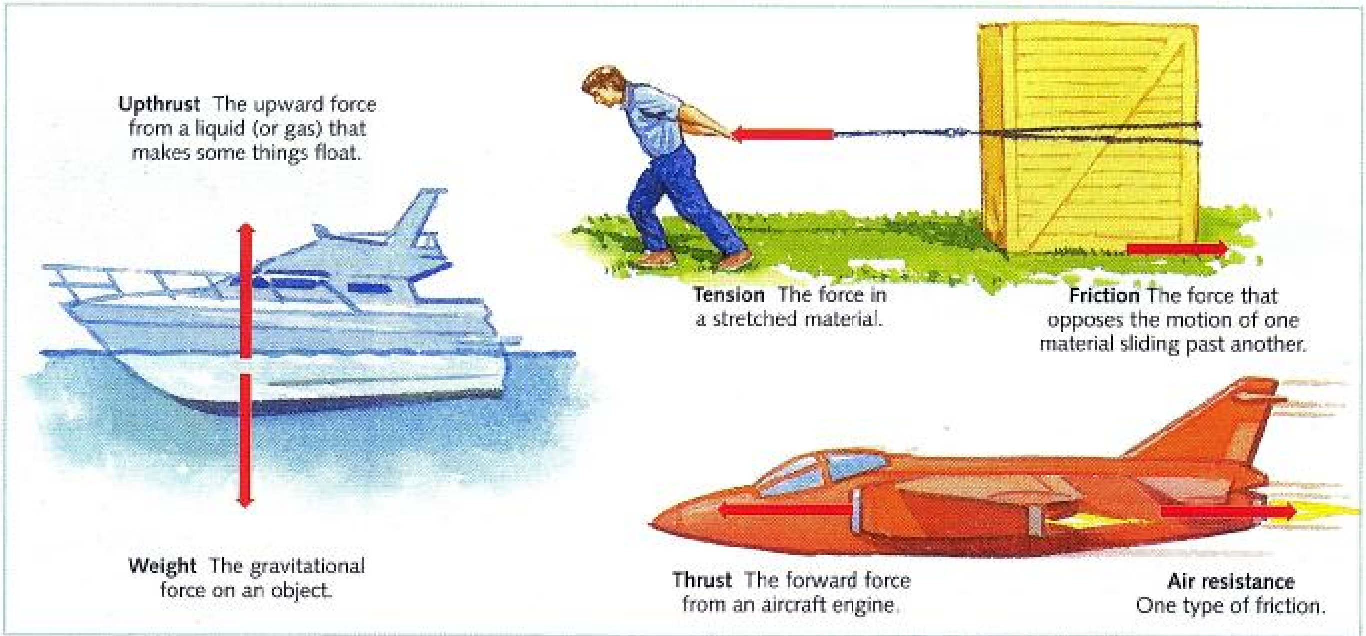
A **force** is a push or a pull, exerted by one object on another. It has direction as well as magnitude (size), so it is a vector.

The SI unit of force is the **newton** (**N**). Small forces can be measured using a spring balance like the one below. The greater the force, the more the spring is stretched and the higher the reading on the scale:



Common forces

Here are some examples of forces:



Deep in space with no forces to slow it, a moving object will keep moving forever.

Motion without force

On Earth, unpowered vehicles soon come to rest because of friction. But with no friction, gravity, or other external force on it, a moving object will keep moving for ever – at a steady speed in a straight line. It doesn't need a force to keep it moving.

This idea is summed up in a law first put forward by Sir Isaac Newton in 1687:

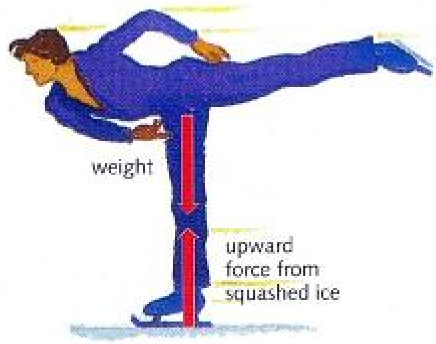
> If no external force is acting on it, an object will
> – if stationary, remain stationary
> – if moving, keep moving at a steady speed in a straight line.

This is known as **Newton's first law of motion**.

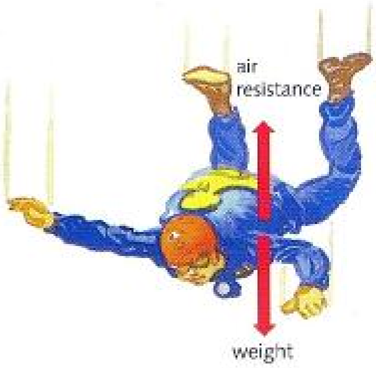
Balanced forces

An object may have several forces on it. But if the forces are in balance, they cancel each other out. Then, the object behaves as if there is no force on it at all. Here are some examples:

Stationary gymnast



Skater with steady velocity



Skydiver with steady velocity

With balanced forces on it, an object is *either* at rest, *or* moving at a steady velocity (steady speed in a straight line). That follows from Newton's first law.

Terminal velocity

When a skydiver falls from a hovering helicopter, as her speed increases, the air resistance on her also increases. Eventually, it is enough to balance her weight, and she gains no more speed. She is at her **terminal velocity**. Typically, this is about 60 m/s, though the actual value depends on air conditions, as well as the size, shape, and weight of the skydiver.

When the skydiver opens her parachute, the extra area of material increases the air resistance. She loses speed rapidly until the forces are again in balance, at a greatly reduced terminal velocity.

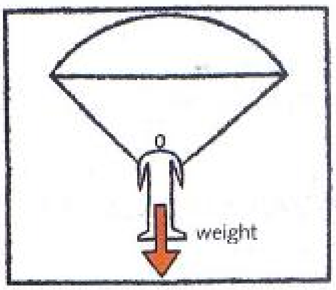
If air resistance balances her weight, why doesn't a skydiver stay still? If she wasn't moving, there wouldn't be any air resistance. And with only her weight acting, she would gain velocity.

Surely, if she is travelling downwards, her weight must be greater than the air resistance? Only if is she is gaining velocity. At a steady velocity, the forces must be in balance. That follows from Newton's first law.


If a skydiver is falling at a steady velocity, the forces on her are balanced: her weight downwards is exactly matched by the air resistance upwards.

Q

1 What is the SI unit of force?
2 What does Newton's first law of motion tell you about the forces on an object that is **a)** stationary **b)** moving at a steady velocity?
3 The parachutist on the right is descending at a steady velocity.
a) What name is given to this velocity?
b) Copy the diagram. Mark in and label another force acting.
c) How does this force compare with the weight?
d) If the parachutist used a larger parachute, how would this affect the steady velocity reached? Explain why.



Related topics: weight and mass **2.09**; floating **3.08**; friction and moving vehicles **4.04**

2.08 Force, mass, and acceleration

Inertia and mass

▶ Once a massive tanker like this is moving it is extremely difficult to stop.

If an object is at rest, it takes a force to make it move. If it is moving, it takes a force to make it go faster, slower, or in a different direction. So all objects resist a change in velocity – even if the velocity is zero. This resistance to change in velocity is called **inertia**. The more mass something has, the more inertia it has.

Velocity is speed in a particular direction.

Any change in velocity is an acceleration. So the more mass something has, the more difficult it is to make it accelerate.

These two forces...

are equivalent to a single force of (5–3) N....

This is the **resultant force**

Resultant force

In the diagram on the left, the two forces are unbalanced. Together, they are equivalent to a single force. This is called the **resultant force**.

If forces are balanced, the resultant force is zero and there is no acceleration. Any other resultant force causes an acceleration – in the same direction as the resultant force.

Linking force, mass, and acceleration

There is a link between the resultant force acting, the mass, and the acceleration produced. For example:

If this resultant force...	acts on this mass...	then this is the acceleration...
1 N	1 kg	1 m/s^2
2 N	2 kg	1 m/s^2
4 N	2 kg	2 m/s^2
6 N	2 kg	3 m/s^2

In all cases, the following equation applies:

$$\text{resultant force} = \text{mass} \times \text{acceleration}$$

In symbols: $F = ma$

This relationship between force, mass, and acceleration is sometimes called **Newton's second law of motion**.

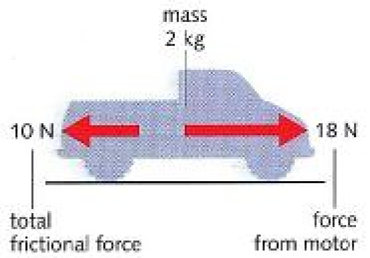
Symbols and units

F = force, in newtons (N)

m = mass, in kilograms (kg)

a = acceleration, in metres/second^2 (m/s^2)

Example What is the acceleration of the model car on the right?



First, work out the resultant force on the car. A force of 18 N to the *right* combined with a force of 10 N to the *left* is equivalent to a force of (18 - 10) N to the *right*. So the resultant force is 8 N.

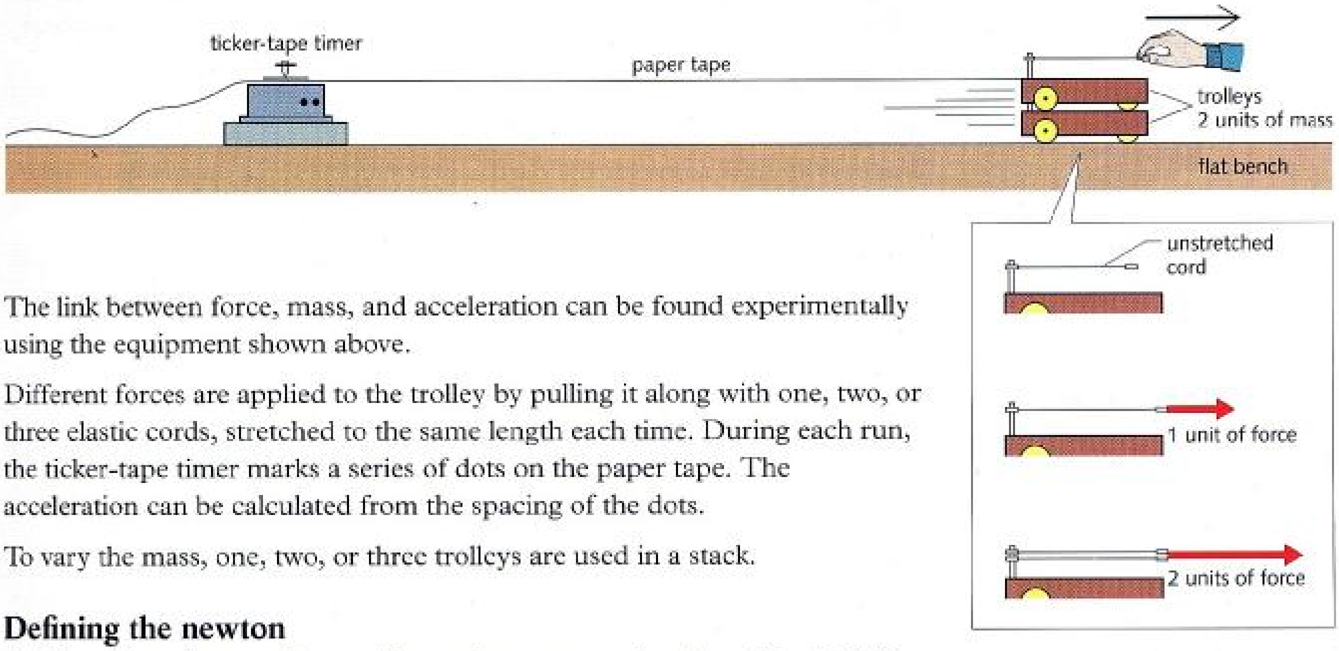
Next, work out the acceleration when $F = 8$ N and $m = 2$ kg:

$F = ma$

So: $8 = 2a$ (omitting units for simplicity)

Rearranged, this gives $a = 4$. So the car's acceleration is 4 m/s².

Finding the link



The link between force, mass, and acceleration can be found experimentally using the equipment shown above.

Different forces are applied to the trolley by pulling it along with one, two, or three elastic cords, stretched to the same length each time. During each run, the ticker-tape timer marks a series of dots on the paper tape. The acceleration can be calculated from the spacing of the dots.

To vary the mass, one, two, or three trolleys are used in a stack.

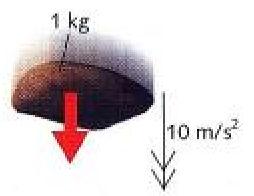
Defining the newton

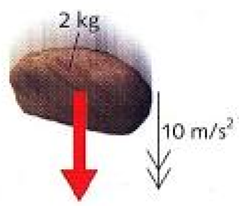
A 1 N resultant force acting on 1 kg produces an acceleration of 1 m/s². This simple result is no accident. It arises from the way the newton is defined:

1 newton is the force required to give a mass of 1 kilogram an acceleration of 1 m/s².

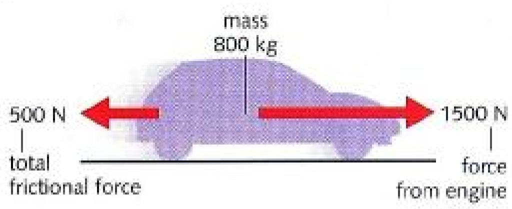
Q

1 a) What equation links resultant force, mass, and acceleration?
b) Use this equation to calculate the resultant force on each of the stones shown below.



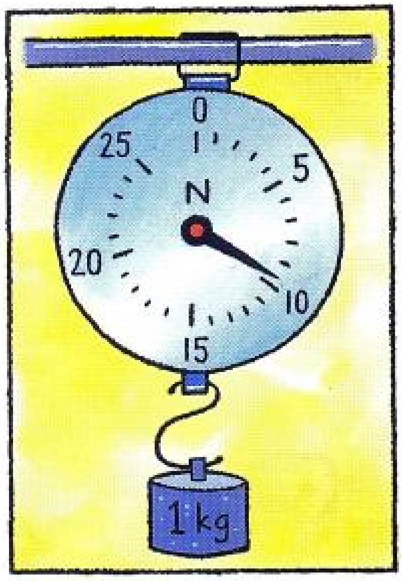


2 a) What is the resultant force on the car below?
b) What is the car's acceleration?
c) If the total frictional force rises to 1500 N, what happens to the car?



Related topics: mass **1.02**; acceleration **2.01**; using ticker-tape **2.04**; balanced forces **2.07**; Newton's second law of motion **2.12**

2.09 Force, weight, and gravity



Near the Earth's surface, a 1 kg mass has a gravitational force on it of about 10 newtons. This is its weight.

Gravitational force

If you hang an object from a spring balance, you measure a downward pull from the Earth. This pull is called a **gravitational force**.

No one is sure what causes gravitational force, but here are some of its main features:

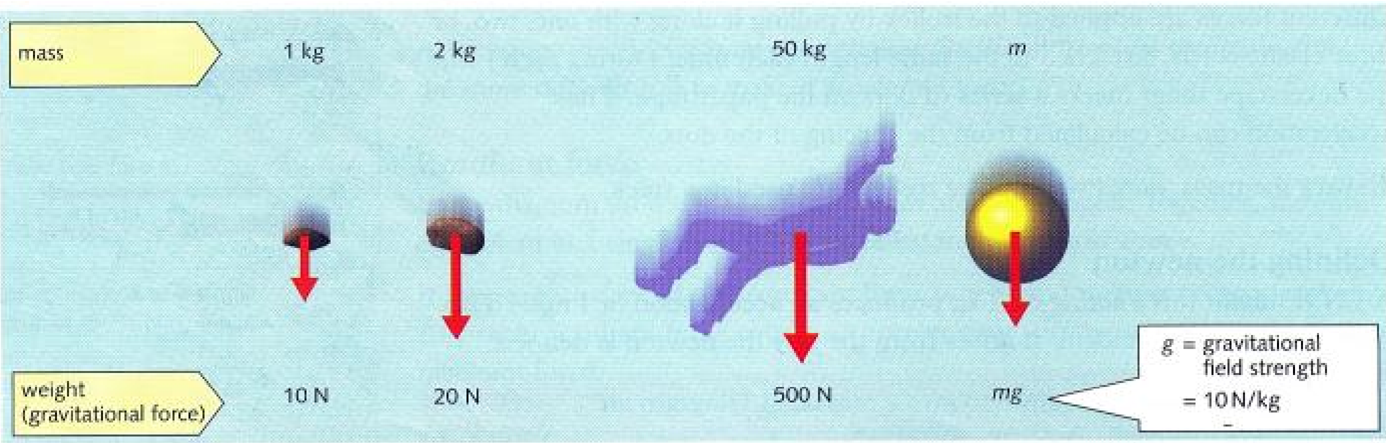
- *All* masses attract each other.
- The greater the masses, the stronger the force.
- The closer the masses, the stronger the force.

The pull between small masses is extremely weak. It is less than 10^{-7} N between you and this book! But the Earth is so massive that its gravitational pull is strong enough to hold most things firmly on the ground.

Weight

Weight is another name for the Earth's gravitational force on an object. Like other forces, it is measured in newtons (N).

Near the Earth's surface, an object of mass 1 kg has a weight of 9.8 N, though 10 N is accurate enough for many calculations and will be used in this book. Greater masses have greater weights. Here are some examples:



Gravitational field strength, *g*

The Earth has a **gravitational field** which exerts a force on any mass in it. Near the Earth's surface, there is a gravitational force of 10 newtons on each kilogram of mass: the Earth's **gravitational field strength** is 10 newtons per kilogram (N/kg).

Gravitational field strength is represented by the symbol ***g***. So:

$$\text{weight} = \text{mass} \times g \qquad (g = 10\ \text{N/kg})$$

In symbols: $W = mg$

In everyday language, we often use the word 'weight' when it should be 'mass'. Even balances, which detect weight, are normally marked in mass units. But the person in the diagram above doesn't 'weigh' 50 kilograms. He has a *mass* of 50 kilograms and a *weight* of 500 newtons.

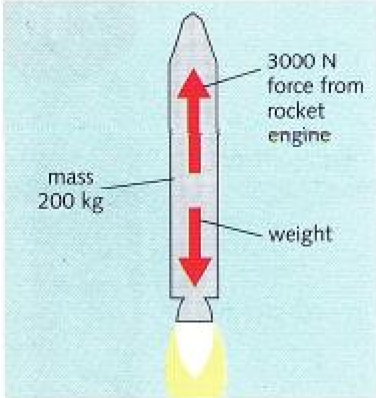
Symbols and units

W = weight, in newtons (N)

m = mass, in kilograms (kg)

g = gravitational field strength, 10 N/kg near the Earth's surface

Example What is its acceleration of the rocket on the right?



To find the acceleration, you need to know the resultant force on the rocket. And to find that, you need to know the rocket's weight:

$$\text{weight} = mg = 200\,\text{kg} \times 10\,\text{N/kg} = 2000\,\text{N}$$

So: resultant force (upwards) $= 3000\,\text{N} - 2000\,\text{N} = 1000\,\text{N}$

But: resultant force = mass × acceleration

So: $1000\,\text{N} = 200\,\text{kg} \times$ acceleration

Rearranged, this gives: acceleration $= 5\,\text{m/s}^2$

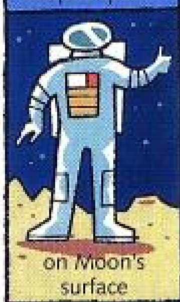
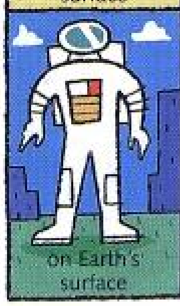
Changing weight, fixed mass

On the Moon, your weight (in newtons) would be less than on Earth, because the Moon's gravitational field is weaker.

Even on Earth, your weight can vary slightly from place to place, because the Earth's gravitational field strength varies. Moving away from the Earth, your weight decreases. If you could go deep into space, and be free of any gravitational pull, your weight would be zero.

Whether on the Earth, on the Moon, or deep in space, your body always has the same resistance to a change in motion. So your mass (in kg) doesn't change – at least, not under normal circumstances. But...

According to Einstein's theory of relativity, mass *can* change. For example, it increases when an object gains speed. However, the change is far too small to detect at speeds much below the speed of light. For all practical purposes, you can assume that mass is constant.

	mass	weight
	100 kg	zero
	100 kg	160 N
	100 kg	1000 N

Two meanings for *g*

In the diagram opposite, the acceleration of each object can be worked out using the equation force = mass × acceleration. For example, the 2 kg mass has a 20 N force on it, so its acceleration is $10\,\text{m/s}^2$.

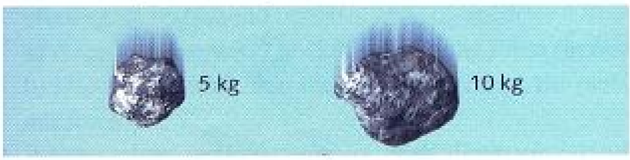
You get the same result for all the other objects. In each case, the acceleration works out at $10\,\text{m/s}^2$, or g (where g is the Earth's gravitational field strength, 10 N/kg).

So g has *two* meanings:

- g is the gravitational field strength (10 newtons per kilogram).
- g is the acceleration of free fall (10 metres per second^2).

Q

Assume that $g = 10$ N/kg and there is no air resistance.



1 The rocks above are falling near the Earth's surface.
  a) What is the weight of each rock?
  b) What is the acceleration of each rock?
  c) What is the gravitational field strength?

2 A spacecraft travels from Earth to Mars, where the gravitational field strength near the surface is 3.7 N/kg. The spacecraft is carrying a probe which has a mass of 100 kg when measured on Earth.
  a) What is the probe's weight on Earth?
  b) What is the probe's mass in space?
  c) What is the probe's mass on Mars?
  d) What is the probe's weight on Mars?
  e) When the probe is falling, near the surface of Mars, what is its acceleration?

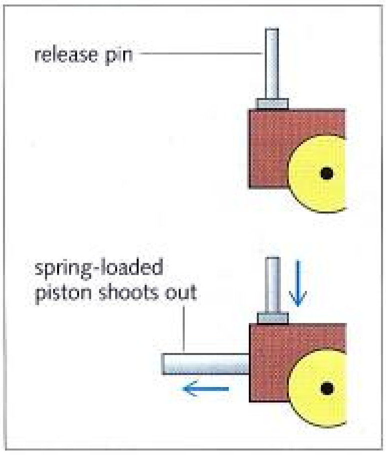
Related topics: acceleration of free fall **2.05**; resultant force and acceleration **2.08**;

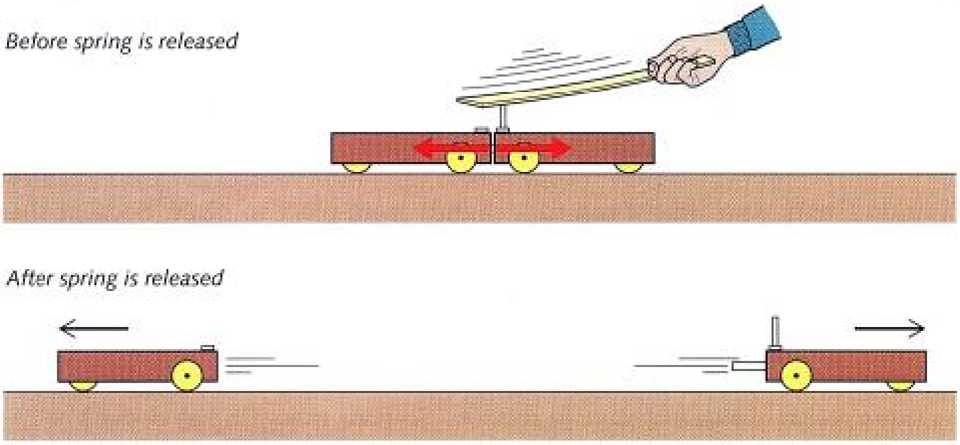
2.10 Action and reaction

Action–reaction pairs

A single force cannot exist by itself. Forces are always pushes or pulls between *two* objects. So they always occur in pairs.

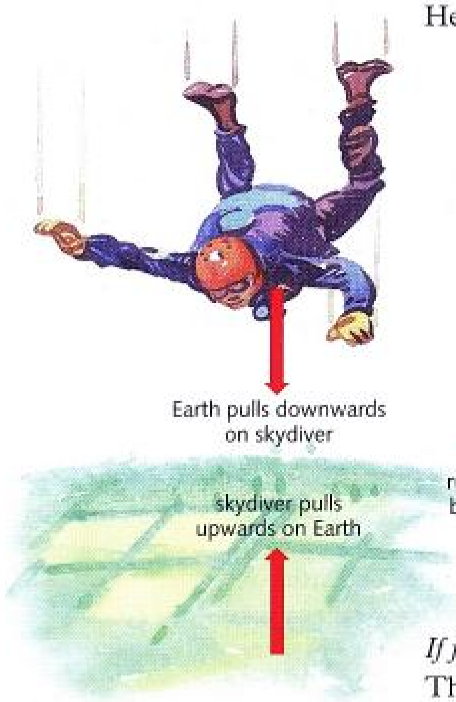
The experiment below shows the effect of a pair of forces. To begin with, the two trolleys are stationary. One of them contains a spring-loaded piston which shoots out when a release pin is hit.

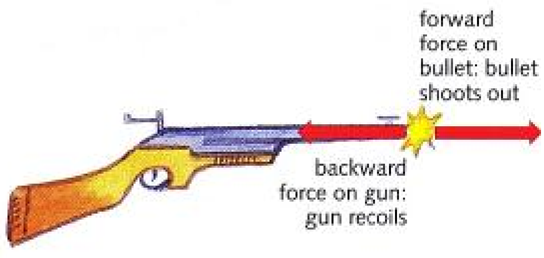




When the piston is released, the trolleys shoot off in opposite directions. Although the piston comes from one trolley only, two equal but opposite forces are produced, one acting on each trolley. The paired forces are known as the **action** and the **reaction**, but it doesn't matter which you call which. One cannot exist without the other.

Here are some more examples of action–reaction pairs:





If forces always occur in pairs, why don't they cancel each other out?
The forces in each pair act on *different* objects, not the same object.

If a skydiver is pulled downwards, why isn't the Earth pulled upwards?
It is! But the Earth is so massive that the upward force on it has far too small an effect for any movement to be detected.

Newton's third law of motion

Sir Isaac Newton was the first to point out that every force has an equal but opposite partner acting on a different object. This idea is summed up by **Newton's third law of motion**:

If object A exerts a force on object B, then object B will exert an equal but opposite force on object A.

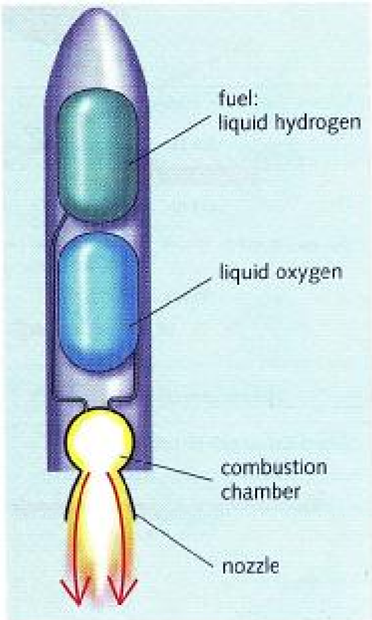
Here is another way of stating the same law:

To every action there is an equal but opposite reaction.

Rockets and jets

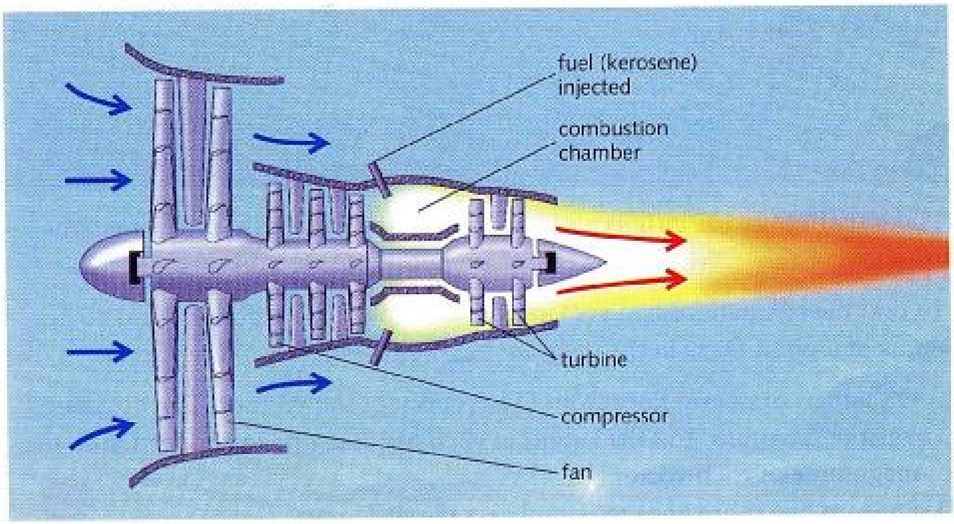
Rockets use the action–reaction principle. A rocket engine gets thrust in one direction by pushing out a huge mass of gas very quickly in the opposite direction. The gas is produced by burning fuel and oxygen. These are either stored as cold liquids, or the fuel may be stored in chemical compounds which have been compressed into solid pellets.

How can a rocket accelerate through space if there is nothing for it to push against? It *does* have something to push against – the huge mass of gas from its burning fuel and oxygen. Fuel and oxygen make up over 90% of the mass of a fully loaded rocket.



▲ A rocket engine. In the combustion chamber, a huge mass of hot gas expands and rushes out of the nozzle. The gas is produced by burning fuel and oxygen.

Jet engines also get thrust by pushing out a huge mass of gas. But the gas is mostly air that has been drawn in at the front:



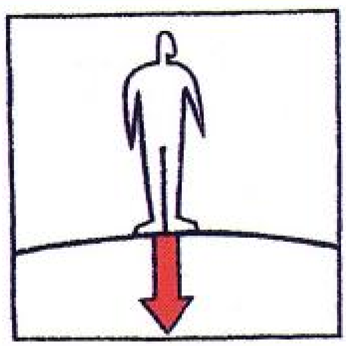
◀ A jet engine. The big fan at the front pushes out a huge mass of air. However, some of the air doesn't come straight out. It is compressed and used to burn fuel in a combustion chamber. As the hot exhaust gas expands, it rushes out of the engine, pushing round a turbine as it goes. The spinning turbine drives the fan and the compressor.

Q

1 The person on the right weighs 500 N. The diagram shows the force of his feet pressing on the ground.
   a) Copy the diagram. Label the size of the force (in newtons).
   b) Draw in the force that the ground exerts on the person's feet. Label the size of this force.

2 When a gun is fired, it exerts a forward force on the bullet. Why does the gun recoil backwards?

3 In the diagram on the opposite page, the forces on the runner and on the ground are equal. Why does the runner move forwards, yet the ground apparently does not move backwards?



Related topics: force **2.07**; gravitational force **2.09**

Revision Checklist

Photocopy the list of topics below and tick the boxes of the ones that are included in your examination syllabus. (Your teacher should be able to tell you which they are.) Use your list when you revise. The spread number in brackets tells you where to find more information.

- ❑ **1** Measuring speed. (2.01)
- ❑ **2** The difference between speed and velocity. (2.01)
- ❑ **3** How acceleration is defined. (2.01)
- ❑ **4** The meaning of retardation. (2.01)
- ❑ **5** Representing motion using distance-time and speed-time graphs. (2.02)
- ❑ **6** Calculating
  - speed from a distance-time graph.
  - acceleration from a speed-time graph.
  - distance travelled from a speed-time graph. (2.02)
- ❑ **7** The equations linking s, u, v, a, and t, and how to use them. (2.03)
- ❑ **8** Recording the motion of a trolley using ticker-tape, and calculating an acceleration from the data collected. (2.04)
- ❑ **9** The acceleration of free fall, g. (2.05)
- ❑ **10** Measuring g. (2.05)
- ❑ **11** Carrying out calculations on the motion of an object in free fall using the s, u, v, a, and t, equations. (2.06)
- ❑ **12** The motion of an object that is falling while travelling sideways. (2.06)
- ❑ **13** Measuring force. (2.07)
- ❑ **14** How an object moves if the forces on it are balanced: Newton's first law of motion. (2.07)
- ❑ **15** Terminal velocity. (2.07)
- ❑ **16** The meaning of resultant force. (2.08)
- ❑ **17** The link between force, mass, and acceleration: Newton's second law of motion. (2.08)
- ❑ **18** Defining the SI unit of force: the newton. (2.08)
- ❑ **19** The properties of gravitational force. (2.09)
- ❑ **20** The difference between weight and mass. (2.09)
- ❑ **21** Defining gravitational field strength. (2.09)
- ❑ **22** Why the weight of an object can vary but, under normal circumstances, the mass does not change. (2.09)
- ❑ **23** Two meanings of g. (2.09)
- ❑ **24** How all forces exist in pairs: Newton's third law of motion. (2.10)
- ❑ **25** How rockets and jets produce a force. (2.10)
- ❑ **26** The difference between vectors and scalars. (2.11)
- ❑ **27** Adding vectors using the parallelogram rule. (2.11)
- ❑ **28** Resolving a vector into two components. (2.11)
- ❑ **29** Defining momentum. (2.12)
- ❑ **30** The link between force and momentum. (2.12)
- ❑ **31** The meaning of impulse. (2.12)
- ❑ **32** The law of conservation of momentum. (2.13)
- ❑ **33** Applying the law of conservation of momentum to objects which are springing apart or colliding. (2.13)
- ❑ **34** The meaning of centripetal force. (2.14)
- ❑ **35** The factors on which centripetal force depends. (2.14)
- ❑ **36** Why an object moving in a circle has an inward acceleration. (2.14)
- ❑ **37** The conditions required for a satellite to move in a circular orbit around the Earth. (2.14)

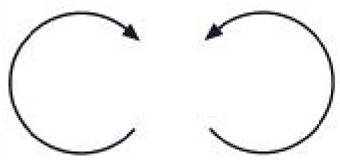
Conditions for equilibrium

If an object is in equilibrium, the forces on it must balance as well as their turning effects. So:

- The sum of the forces in one direction must equal the sum of the forces in the opposite direction.
- The principle of moments must apply.

For example, in diagram A on the opposite page, the upward force from the support must be 15 N, to balance the 10 N + 5 N total downward force. Also, if you take moments about *any* point – P, for example – the total clockwise moment must equal the total anticlockwise moment.

When taking moments about P, you need to include the moment of the upward force from the support. This doesn't arise when taking moments about O because the force has no moment about that point.



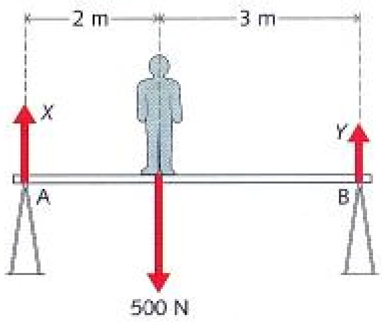
Clockwise .... or anticlockwise?

In the diagram below, the 500 N force has a *clockwise* moment about A, but an *anticlockwise* moment about B.

To decide whether a moment is clockwise or anticlockwise about a point, imagine that the diagram is pinned to the table through the point, then decide which way the force arrow is trying to turn the paper.

Solving a problem

Example Below right, someone of weight 500 N is standing on a plank supported by two trestles. Calculate the upward forces, *X* and *Y*, exerted by the trestles on the plank. (Assume the plank has negligible weight.)



The system is in equilibrium, so the principle of moments applies. You can take moments about any point. But taking moments about A or B gets rid of one of the unknowns, *X* or *Y*.

Taking moments about A:
clockwise moment = 500 N × 2 m = 1000 N m
anticlockwise moment = $Y \times 5$ m
As the moments balance, $5Y$m = 1000 N m
So: $Y = 200$ N

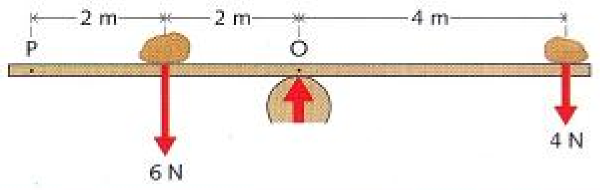
From here, there are two methods of finding X. *Either* take moments about B and do a calculation like the one above. *Or* use the fact that $X + Y$ must equal the 500 N downward force. By either method: $X = 300$ N

Q

1 The moment (turning effect) of a force depends on two factors. What are they?

2 What is the principle of moments? What other rule also applies if an object is in equilibrium?

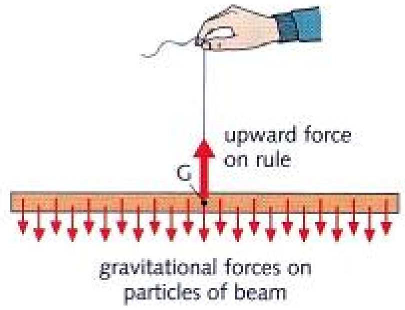
3 Below, someone is trying to balance a plank with stones. The plank has negligible weight.
a) Calculate the moment of the 4 N force about O.
b) Calculate the moment of the 6 N force about O.



c) Will the plank balance? If not, which way will it tip?
d) What extra force is needed at point P to balance the plank?
e) In which direction must the force at P act?

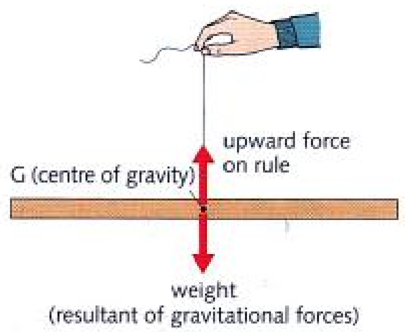
4 In *diagram* **B** on the opposite page:
a) What is the upward force from the support?
b) If moments are taken *about point P*, which forces have clockwise moments? What is the total clockwise moment about P?
c) Which force or forces have anticlockwise moments about P? What is the total anticlockwise moment about P?
d) Comparing moments about P, does the principle of moments apply?

3.02 Centre of gravity



Like other objects, the beam on the left is made up of lots of tiny particles, each with a small gravitational force on it. The beam balances when suspended at one particular point, G, because the gravitational forces have turning effects about G which cancel out.

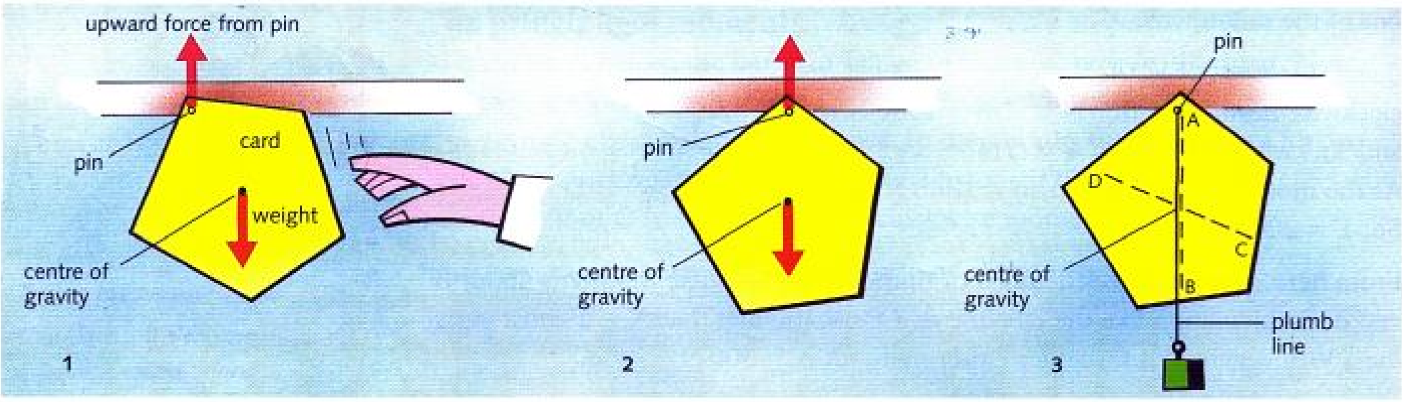
Together, the small gravitational forces act like a single force at G. In other words, they have a resultant at G. This resultant is the beam's **weight**. G is the **centre of gravity** (or **centre of mass**).



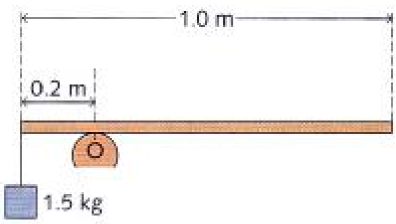
Finding a centre of gravity

In diagram 1 below, the card can swing freely from the pin. When the card is released, the forces on it turn the card until its centre of gravity is vertically under the pin, as in diagram 2. Whichever point the card is suspended from, it will always hang with its centre of gravity vertically under the pin. This fact can be used to find the centre of gravity.

In diagram 3, the centre of gravity lies somewhere along the plumb line, whose position is marked by the line AB. If the card is suspended at a different point, a second line CD can be drawn. The centre of gravity must also lie along this line, so it is at the point where AB crosses CD.



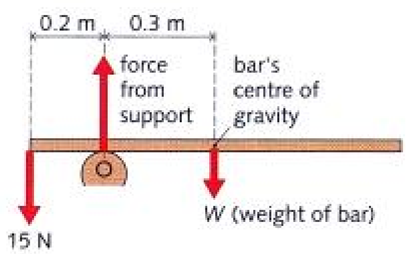
Heavy bar problem



In simple problems, you are often told that a balanced bar has negligible weight. In more complicated problems, you have to include the weight.

Example If a uniform bar balances, as on the left, with a 1.5 kg mass attached to one end, what is its weight? (g = 10 N/kg)

To solve the problem, redraw the diagram to show all the forces and distances, as in the lower diagram. As g = 10 N/kg, the 1.5 kg mass has a weight of 15 N. 'Uniform' means that the bar's weight is evenly distributed, so the centre of gravity of the bar (by itself) is at the mid-point, 0.5 m from one end. The bar's weight W acts at this point.

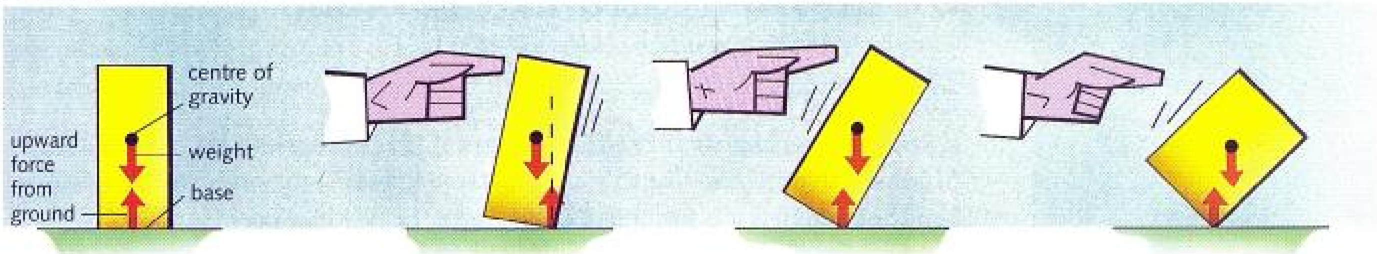


Now take moments about the support, O. The upward force has no moment about this point, but there is an anticlockwise moment of 15 N × 0.2 m and a clockwise moment of $W \times 0.3$ m. As the bar is in equilibrium:

$15\,\text{N} \times 0.2\,\text{m} = W \times 0.3\,\text{m}$

So: the bar's weight W is 10 N.

Stability



This box is in equilibrium. The forces on it are balanced, and so are their turning effects.

With a small tilt, the forces will turn the box back to its original position.

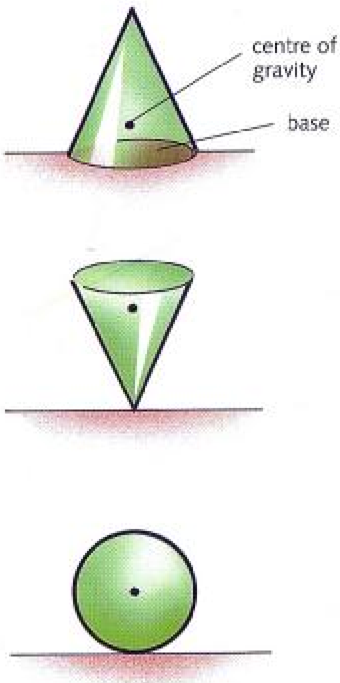
With a large tilt, the forces will tip the box over.

A box with a wider base and a lower centre of gravity can be tilted further before it falls over.

If the box above is pushed a little and then released, it falls back to its original position. Its position was **stable**. If the box is pushed much further, it topples. It starts to topple as soon as its centre of gravity passes over the edge of its base. From then on, the forces on the box have a turning effect which tips it even further. A box with a wider base and/or a lower centre of gravity is more stable. It can be tilted to a greater angle before it starts to topple.

States of equilibrium

Here are three types of equilibrium:



Stable equilibrium If you tip the cone a little, the centre of gravity stays over the base. So the cone falls back to its original position.

Unstable equilibrium The cone is balanced, but only briefly. Its pointed 'base' is so small that the centre of gravity immediately passes beyond it.

Neutral equilibrium Left alone, the ball stays where it is. When moved, it stays in its new position. Wherever it lies, its centre of gravity is always exactly over the point which is its 'base'.

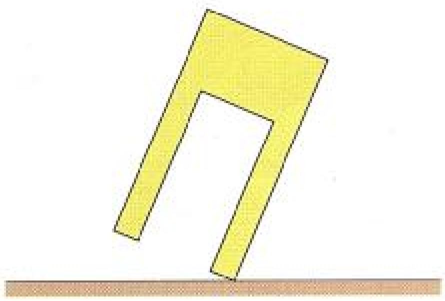
This gymnast will stay balanced – as long as she keeps her centre of gravity over the beam.

Q

1 The stool on the right is about to topple over.
   a) Copy the diagram, showing the position of the centre of gravity.
   b) Give *two* features which would make the stool more stable.

2 A uniform metre rule has a 4 N weight hanging from one end. The rule balances when suspended from a point 0.1 m from that end.
   a) Draw a diagram to show the rule and the forces on it.
   b) Calculate the weight of the rule.

3 Draw diagrams to show a drawing pin in positions of stable, unstable, and neutral equilibrium.



Related topics: resultant force **2.08** and **2.11**; mass, weight, and *g* **2.09**; turning effects, moments, and equilibrium **3.01**