

EXPONENTIAL FUNCTIONS

Exponential function with the base a is given with the notation $f(x): y = a^x$, while a is the *exponent* and it is a positive real number that doesn't equal 1, i.e. $a \in (0,1) \cup (1,\infty)$. The graph of an exponential function is so called *exponential curve* and its domain is the set of all real numbers, i.e. $x \in \mathbb{R}$, $D(f) = \mathbb{R}$.

Formula: $y = a^x$; the base $a \in (0,1) \cup (1,\infty)$; the exponent $x \in \mathbb{R}$

Types of exponential functions:

- *Decadic* exponential function: $f(x): y = 10^x$, where the base is number 10
- *Natural* exponential function: $f(x): y = e^x$, where the base e is so called Euler's number, $e = 2,718281\dots$
- *Other* exponential functions whose properties are dependent on their base a :

$a > 1$	$0 < a < 1$
<p>Graph:</p>	<p>Graph:</p>
Domain: $D(f) = \mathbb{R}$	Domain: $D(f) = \mathbb{R}$
Range of values: $V(f) = (0, \infty)$	Range of values: $V(f) = (0, \infty)$
Increasing function \rightarrow one-to-one function	Decreasing function \rightarrow one-to-one function
Bounded from below with 0, not bounded from above	Bounded from below with 0, not bounded from above
no minimum, no maximum	no minimum, no maximum
Neither even, nor odd	Neither even, nor odd
Graph passes through the point (0, 1)	Graph passes through the point (0, 1)

Function and its notation: $y = a^x$ implies that the point $(0, 1)$ has to lie on the graph, in other words point $(0, 1)$ is the point of intersection of the curve with y -axis. Why?

Because if you raise a real base a to the power of $x = 0$, you will always get $y = 1$ ($a^0 = 1$)

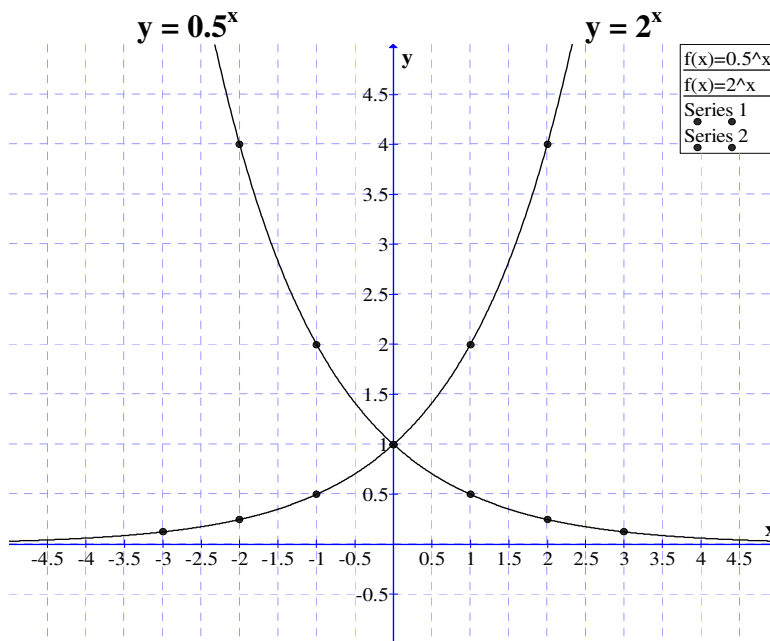
As a function of the *real* variable x , the graph of $y = a^x$ is always positive (above the x axis). It never touches the x axis, although it gets arbitrarily close to it (thus, the x axis is a horizontal asymptote to the graph).

E1 There are two functions: $y = 2^x$ and $y = 0.5^x$. Draw graphs of these functions.

$y = 2^x$							
x	-3	-2	-1	0	1	2	3
2^x	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3
y	0.125	0.25	0.5	1	2	4	8

$y = 0.5^x$							
x	-3	-2	-1	0	1	2	3
0.5^x	0.5^{-3}	0.5^{-2}	0.5^{-1}	0.5^0	0.5^1	0.5^2	0.5^3
y	8	4	2	1	0.5	0.25	0.125

GRAPH:



Ex: Sketch the graphs of the following functions:

1. $y = 2^x + 1$
2. $y = 2^{x+1}$