EQUATIONS

Equation is a notation of equality of two expressions, in which the value of variable needs to be set to reach the true mathematical statement.

The variable in an equation is called **unknown**. The number (value of variable), for which the expressions are equal, is called the **root of the equation.** There are two sides, left and right.

By solving the equation, we may:

- change the sides of an equation
- substitute one side with an expression, which is equal to it
- add an expression, which is defined in the whole definition set, to the both sides
- multiply both sides with an expression, which is defined in the whole definition set
- raise the power of both sides of equation to a natural exponent
- extract the root of both sides, if the sides reach nonnegative values only

LINEAR EQUATIONS

a, b $\in R, a \neq 0$. Then the equation ax + b = 0 is called **linear equation**. Its definition set is R.

The only root of such an equation is number x = $\frac{-b}{a}$.

In one unknown:

- where the unknown is on both sides of the equation: 5c – 4 = 3c – 8
- where brackets are involved: 3(4p +5) = 5 (3p – 4)
- where fractions are involved: $\frac{2}{3}$(w + 3) = 4w - 3

- EQUATIONS WITH THE UNKNOWN IN THE DENOMINATOR

Here it is required that the denominator is different from zero, and therefore we have to state conditions, i.e. to state the domain. Otherwise the expression would not make sense. After stating conditions, we multiply both sides of the equation by the least common multiple of numbers in the denominator to remove the fractions. After solving the problem, we have to compare the result with the conditions and check the correctness.

$$\frac{x+7}{x-5} - 2 = - \frac{5+x}{x-7}$$

EQUATIONS WITH THE ABSOLUTE VALUE/MODULUS

For each real number a it is true that:

if $a \geq 0$ then $|a| = a$

if $a < 0$ then $|a| = -a$, i.e. $a = -5 >$ $|-5| = -(-5) = 5$

For each real number **a** is |a| a nonnegative number. Therefore if **a** is a negative number, **-a** is a positive number.

Equations (inequalities) in which the unknown is in the absolute value are solved through dividing the definition set (domain) into such intervals, in which each of the expression does not change the sign. In such integrals we solve that equation which is equivalent with the original one and does not contain the absolute value. Finally the solutions have to be compared and discussed with the definition intervals.

Solve the problem:

$A(x) = |x-3| + 5$

We have to think of two cases:

$x-3 \geq 0$ then $|x-3| = x-3$ and $A(x) = x+2$

$x-3<0$ then $|x-3| = -x+3$ and $A(x) = 8-x$

SOLVING EQUATIONS GRAPHICALLY

A linear graph can be identified from its equation. The equation of all linear graphs is of the form: $y = \boldsymbol{m}x + \boldsymbol{c}$ where **m** is the gradient and **c** is the intercept with the y-axis.

E.g.: $y=\frac{3}{4}x + 1$ Gradient is $\frac{3}{4}$, and an intercept with the y-axis is at 1. The gradient is positive, therefore the line is sloping upwards from left to right.

$y = -\frac{5}{2}x - 3$ Gradient is $-\frac{5}{2}$ and an intercept with the y-axis is at –3. The gradient is negative, and therefore the line is sloping downwards from left to right.

Being able to identify the gradient and intercept with the y-axis from a linear equation enables you to sketch or draw a graph.

TO **DRAW** A GRAPH:

- draw, graduate, and label the pairs of axes
- plot the point (0, -5), i.e. the y-intercept
- from the y-intercept show the gradient, i.e. to plot a second point on the graph
- draw a straight line passing through the two points
- label the linear graph with its equation
- you **draw** a graph, so be precise and accurate!

You may have the equation in the form of: $3y + 4x = 6$. Then this linear equation needs to be rearranged to put it in the form $y = \boldsymbol{m}x + \boldsymbol{c}$. Note that rearranging equation is not changing the equation; it is producing an *equivalent* equation.

$3y + 4x = 6$

$3y = 6 – 4x$

$y = -\frac{4}{3}x + 2$

INEQUALITIES

The inequality signs:

$>$ greater than

$<$ less than

$\geq$ greater than or equal to

$\leq$ less than or equal to

can be used to define a range of values for a variable. E.g. $–3 < c \leq 4$ means the variable **c** can have any greater value than –3 but less than or equal to 4. This is called the **solution set for** c.

To solve an inequality you can treat it like an equation. The only difference is that if we multiply or divide both sides of an inequality by a negative value the inequality signs will reverse. Do not forget to change the sign then!!!

<u>Inequalities with the absolute value</u>: we have to find out those real numbers x, for which the absolute value is 0. We divide the solution set with this number and we will solve the problem in the subsets similarly as we do while calculating equations.