5.21

MAGNETICALLY INDUCED ELECTRIC CURRENTS

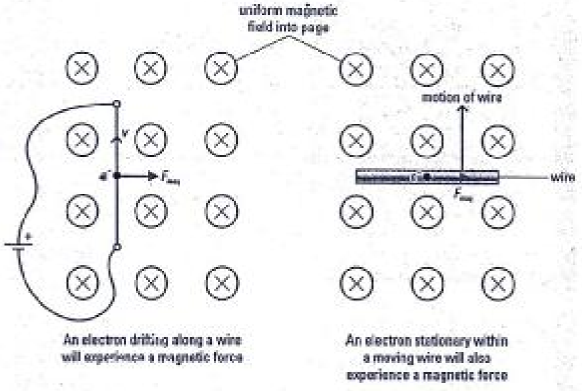
OBJECTIVES

- producing currents in moving conductors
- e.m.f. across a moving wire

The magnetic force on a moving charge can be used as a means of generating an electrical current. Any piece of conducting material contains charges, and simply moving the material through a magnetic field will result in forces being exerted on those charges. We normally think of magnetic forces as acting on currents, but really they act on any moving charge – even the charges within a wire that is moving.

Helpful hint

Of course, the positively charged protons within the moving wire will also experience a force, but this has no effect as they are locked inside the nuclei of the atoms, and the atoms in turn are held in a lattice.



An electron drifting along a wire will experience a magnetic force

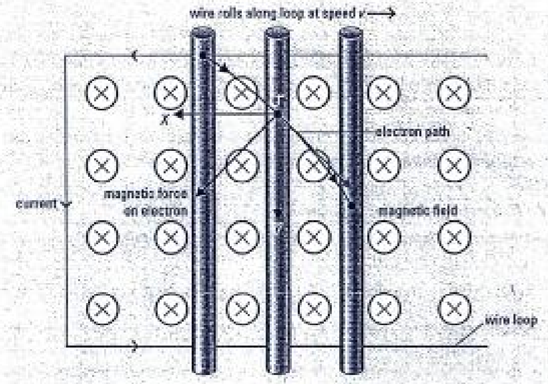
An electron stationary within a moving wire will also experience a magnetic force

Magnetic forces on moving electrons.

Helpful hint

If there was no magnetic field the electrons would be moving with the wire, and so would not collide with the atoms.

In the right-hand part of the diagram above, the magnetic force drives electrons along the wire from left to right as the wire moves up the page. Effectively, there is an electric current along the length of the wire. The electrons collide with atoms as they drift along, so their kinetic energy is converted into thermal energy. The electrons are as much a part of the wire as the atoms within it, so the wire is also losing kinetic energy. This effect is known as **current braking** and can be used in practical situations.



The magnetic force on the electrons in the wire has two components. X is part of the BIl force on the wire; Y drives the electrons down the wire, establishing the current.

If there is some way of removing charge at one end of the wire and replacing it at the other, a continuous electrical current can be maintained in the wire. This effect is being investigated by NASA as a means of generating power for spacecraft in orbit.

As the Space Shuttle orbits with an electrical tether unwound, the force due to the Earth's magnetic field drives electrons along the wire. The positively charged end of the tether will attract negatively charged particles from space (many are trapped in the Earth's magnetic field). The other end of the wire will be negative and will attract positive particles. These particles discharge the ends of the wire, effectively closing the circuit.

Broken tether

Unfortunately the first experiment carried out aboard the Space Shuttle failed. The tether broke as it was being unreeled – a shard of metal accidentally cut through the wire.

In this situation the kinetic energy of the satellite (the orbiting Shuttle) is being converted into thermal energy in the wire (and in any load placed on the circuit).

Electromotive force (e.m.f.) across a moving wire

In the slightly simpler situation of a straight wire moving through a magnetic field, the e.m.f. across the wire can be calculated. A constant current can be maintained by having it roll along a wire loop as in the diagram on the previous page. The magnetic force acting on the electrons has a component perpendicular to the wire which is the BIl force that acts on any current-carrying wire.

$$\text{magnetic force on current} = BIl$$

This force acts in the opposite direction to the motion of the wire, and so an external force, F, is required to keep a constant speed.

$$\therefore \text{ force required to keep a constant speed} = BIl$$

$$\therefore \text{ work done on wire per second} = \text{force} \times \text{distance moved in one second} = BIlv$$

This energy must replace that converted into thermal energy in the wire.

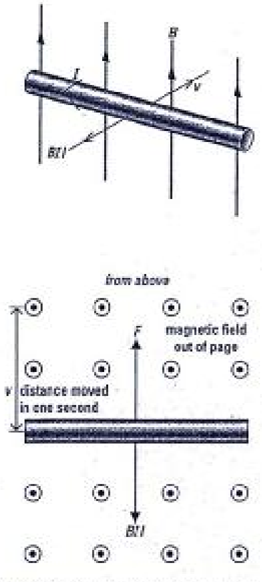
$$\therefore \quad BIlv = I^2R$$

(R is the resistance of the wire), so

$$Blv = IR$$

The left-hand term in this equation represents the conversion of kinetic energy into electrical energy and so is classed as an e.m.f. The right-hand term is the voltage drop across the wire. Hence we can say that the e.m.f. induced in the moving wire is given by

$$E = Blv$$



Calculating the e.m.f across a moving wire.

PRACTICE

1 The length of a train axle is 3 m. The train is moving at 40 km h^{-1} at 90° to a magnetic field of 50 μT. What is the e.m.f. induced across the axle?

2 A conducting wire, 0.5 m long, is moving at 20 m s^{-1} through a magnetic field of 25 μT so that the wire is at 90° to the field lines. The induced current is 250 μA. What is the force required to maintain the constant speed? What is the work done per second by this force? What is the resistance of the wire?

3 An orbiting Space Shuttle is travelling at 3100 m s^{-1}. It extends a 20 km long conducting tether so that the line of the tether is perpendicular to the Earth's surface. A current of 5 A is measured in the tether and the power generated is 15 kW. Estimate the average magnetic field strength along the length of the tether.

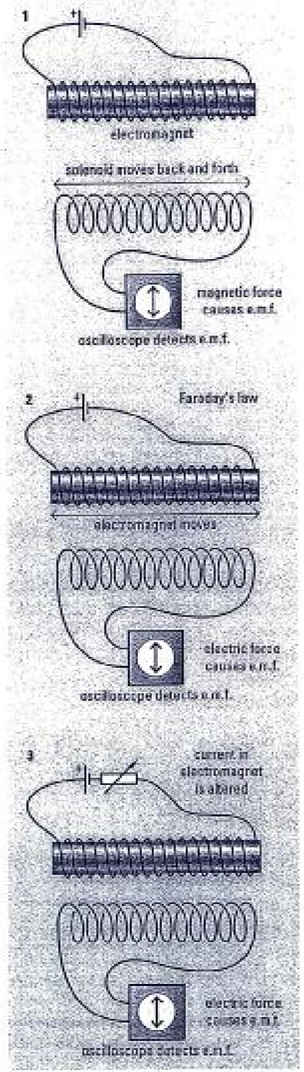
4 By making reasonable estimations of the required numbers, estimate the e.m.f. induced between the tips of a passenger aeroplane's wings while flying through the Earth's magnetic field (50 μT perpendicular to the wing's surface). Is this likely to be dangerous?

5.22

ELECTRICALLY INDUCED CURRENTS

OBJECTIVES

- induced electric fields
- magnetic flux – flux linked, flux cut
- Faraday's law of induction

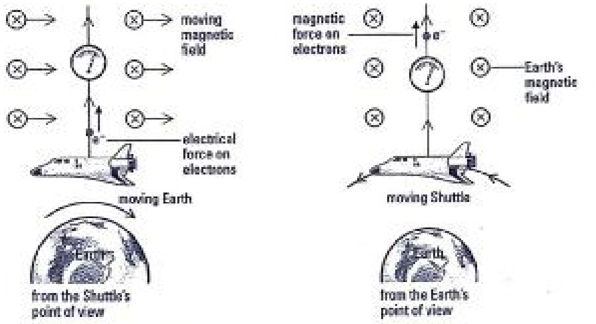


Three experiments that demonstrate induction effects. An experimenter who could only see the oscilloscope would not be able to tell which of the three experiments was taking place.

In the previous spread a plan for producing an electric current in a tether extending from a Space Shuttle was described. Viewed from Earth, this effect is due to the magnetic force on the charges within the tether deflecting their motion along its length. Provided the ends of the wire are discharged (by attracting charged particles from space), a constant current can be maintained.

However, from the astronaut's point of view this explanation will not do. They see a stationary wire and so no magnetic forces can be acting on the charges within it. Yet they can measure a current in the tether.

The astronauts are observing the tether in a **frame of reference** in which it is stationary and the magnetic field is moving. People on Earth are observing from another frame of reference in which the tether is moving through a magnetic field. Both frames of reference must observe the same physical effect – an e.m.f. is acting. From one frame of reference that e.m.f. is due to a magnetic force; from the other frame of reference it is due to an **induced electric force.**



Observing the Space Shuttle tether experiment from two equivalent frames of reference.

Induced electric fields

This effect can be demonstrated in a laboratory. There are three possibilities:

1 Imagine holding an electromagnet still and moving a solenoid back and forth through the field. The magnetic force causes a current in the solenoid, and the e.m.f. can be detected by connecting the leads from the solenoid to an oscilloscope. The spot on the oscilloscope is deflected up and down as the electromagnet moves.

2 Now imagine keeping the solenoid still and moving the electromagnet back and forth. There can be no magnetic force acting, but the oscilloscope spot still moves. The two situations are equivalent to each other. Someone moving back and forth with the electromagnet would see it as being still and the solenoid moving – exactly the same as the previous experiment.

3 A final variation is a stationary solenoid in a magnetic field that is changing in strength. Nothing is moving, yet the oscilloscope will detect an e.m.f. in the solenoid. From the solenoid's point of view, all that is happening is that the magnetic field passing through it is changing. The change is not because the magnet producing the field is getting *nearer* (as before); this time the magnet is staying in the same place but is getting *stronger*.

In **1** the e.m.f. is produced by a magnetic field. In **2** and **3** the e.m.f. is due to an *induced electric field*. The strength of the induced electric field depends on the rate at which the magnetic field is changing. In order to quantify this it is helpful to introduce the **flux** of a magnetic field.

Magnetic flux

Definition: magnetic flux

$$\left(\begin{array}{c}\text{magnetic flux in area } A\\ \text{of a magnetic field (Wb)}\end{array}\right) = \left(\begin{array}{c}\text{average component of}\\ B \text{ at } 90^\circ \text{ to area (T)}\end{array}\right) \times \text{area } A\ (\text{m}^2)$$

$$\phi = B_\perp \times A$$

The unit of flux is the weber (Wb), and 1 Wb = 1 T m^2.

- The **flux linked** to a coil ($N\phi$) with multiple turns is the flux through one turn (ϕ) multiplied by the number of turns (N).
- The **flux cut** by a moving wire is the magnetic field strength multiplied by the area swept out by the wire.

Worked example 1

A 100 cm^2 wire loop is placed in a uniform magnetic field that passes through the plane of the loop at 90°. The magnetic field strength is 0.03 T. What is the flux through to the loop?

flux, $\phi = (0.03\ \text{T}) \times (100 \times 10^{-4}\ \text{m}^2) = 3 \times 10^{-4}$ Wb

Worked example 2

What is the flux linked to the coil shown in the diagram on the top right?

$B_\perp = B\cos\alpha \quad \therefore$ flux linked $N\phi = BAN\cos\alpha$

Worked example 3

A wire of length l is moving horizontally at constant speed v in a direction at 90° to its length. The wire is passing through a vertical magnetic field of uniform strength B. What is the flux cut by the wire per second?

In one second, the wire moves a distance v, and the area swept out = lv.

$\therefore$ flux cut by the wire per second = Blv

The e.m.f. induced in a circuit can be calculated from **Faraday's law of Induction.**

Faraday's law of induction

$$\text{e.m.f.} = -\text{ rate of change of flux} = -\frac{\Delta\phi}{\Delta t}$$

Or in calculus notation for the instantaneous rate of change of the flux:

$$\varepsilon = -\frac{d\phi}{dt}$$

The minus sign indicates the direction in which the e.m.f. is induced (see above right).

Worked example 4

What is the e.m.f. across the moving wire from worked example 3?

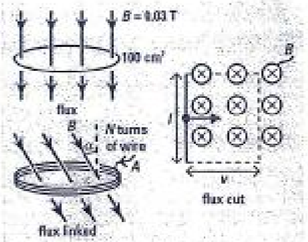
e.m.f. = – rate of change of flux = – flux cut per second = $-Blv$

Note: this is the same answer that we obtained previously using the magnetic force acting on the charges.

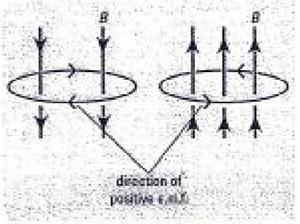
Worked example 5

The magnetic field passing through the 100 cm^2 coil in worked example 1 reduces from 0.03 T to 0.01 T in 20 s. What is the e.m.f. induced in the coil?

$$\text{e.m.f.} = -\text{ rate of change of flux} = -\frac{(0.01\ \text{T}) - (0.03\ \text{T})}{20\ \text{s}} = 1\ \text{mV}$$



Flux, flux linked, and flux cut are essentially the same physical ideas in slightly different contexts.



The convention for the direction of positive e.m.f.: if the magnetic field lines are pointing down into the page when viewed from above, a positive e.m.f. will be in a clockwise direction round a wire loop. In the case of a moving wire, the direction is taken round the perimeter of the area swept out.

PRACTICE

1 A flat loop of wire of area 50 cm^2 has a uniform magnetic field of 0.05 T passing through the plane of the loop at 90°. The loop is now rotated slowly until the loop lies parallel to the field lines. Sketch the situation before and after the rotation of the loop. If the loop takes 30 s to get to its new position, what is the e.m.f. induced in the loop while it is moving?

2 A 50 turn coil of wire of area 4 cm^2 is placed inside an electromagnet. The magnetic field strength is 0.03 T and passes through the coil at 90°. What is the flux linked to the coil? The electromagnet is turned off and the field drops uniformly to zero in 0.12 s. What is the e.m.f. induced in the coil?

5.23

ELECTROMAGNETIC INDUCTION

OBJECTIVES

- Lenz's law
- current braking
- eddy currents

Faraday's law

Faraday's law is a remarkable piece of physics because it links three physically different effects with the one equation:

$$\text{e.m.f.} = -\text{ rate of change of flux} = -\frac{d\phi}{dt}$$

1 $\frac{d\phi}{dt}$ can be the flux cut by a moving wire.

2 $\frac{d\phi}{dt}$ can be the change in flux due to a moving magnet.

3 $\frac{d\phi}{dt}$ can be the change in flux due to a stationary magnet which is changing in strength.

Faraday's contribution was to experimentally demonstrate the equivalence of **2** and **3**. The law of induction does not identify what physical effect is behind the e.m.f. (in **1** it is a magnetic force; in **2** and **3** it is an induced electric field), but it guarantees that however one looks at the situation the same physical result will always be seen.

Non-conservative fields

Induced electric fields are not conservative. The work done in moving a charge round a loop in a conservative field is zero. An induced field can drive a current round a loop of wire. To do this requires energy as the current heats the wire, and so induced electric fields can not be conservative. This is a direct consequence of them not being produced by charges.

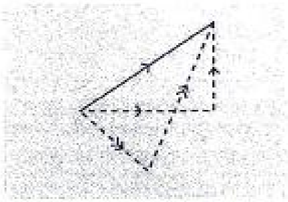
Induction and relativity

It seems magical that an induced electric field can spring up in one frame of reference when there is a moving or changing magnetic field. Unlike any other electric field an induced field does not have electric charges acting as its source. The field lines of induced fields form closed loops – like magnetic field lines always do. Einstein's powerful physical intuition worked on the idea that a magnetic field in one frame of reference could appear as an electric field in another, and the result was the **special theory of relativity**.

The link between electric and magnetic fields suggests that there is only one *electromagnetic field* which interacts with stationary and moving charges. Any physical effect can be explained by an electric force, a magnetic force, or a mixture of both depending on the frame of reference from which the experiment is being viewed.

Think of this as being rather like the components of a vector.

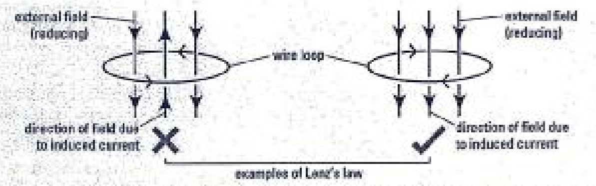
- Any vector can be resolved into components, and there are many ways of doing this depending on the axes chosen.
- The electromagnetic field can be split into electric and magnetic 'components' in different combinations, depending on the frame of reference in which the field is viewed.



A vector can be resolved into components in many different ways.

Lenz's law

The induced current is always in a direction that will help to counteract the change in flux that is producing it.



Lenz's law implies that the field due to an induced current must cancel the change in the field that is producing it.

Lenz's law

The minus sign in Faraday's law shows the direction in which the e.m.f. is produced. It is expressing a law of physics discovered in 1834 by the Russian physicist Emil Lenz.

In the diagram above, the magnetic field strength through the loop is being steadily *reduced*. The total magnetic field in the loop is made up of two parts – the external magnetic field (produced in some way not shown on the diagram) and the magnetic field due to the induced current. The induced current could either circulate clockwise or anticlockwise, and Lenz's law helps to decide which is correct.

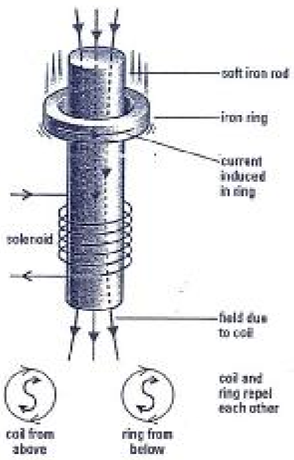
Anticlockwise The field of the induced current is in the *opposite* direction to the external field. The two fields tend to *cancel* inside the loop. This would *reduce* the flux linked, further inducing a greater current; but a greater current has a greater magnetic field, which improves the cancellation, and the problem gets worse. This situation would violate conservation of energy, as the current would increase almost without limit.

Clockwise The field of the induced current *adds* to that of the external field, partly compensating for its reduction, and this tends to *reduce* the size of the induced current, rather than increasing it. The magnetic field of the induced current can never completely replace the lost external field, so there is some flux loss, but the situation does *not* violate conservation of energy.

These examples show that Lenz's law is effectively a restatement of conservation of energy.

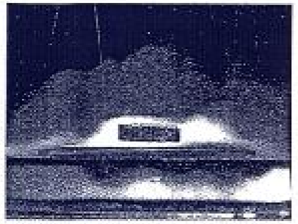
Current braking

A metal ring falling towards a solenoid will experience a braking force that slows its fall. As the ring is falling towards the solenoid, the magnetic field strength and so the flux linked to the ring will be increasing, and so a current is induced in the ring. Lenz's law shows that the current will circulate in the ring in the direction that allows the magnetic field of the current to compensate for the change in flux. The ring acts like a small coil, magnetized with the opposite pole facing downwards, and so is repelled by the solenoid.



Current braking.

The induced currents will not be large enough to cause the ring to hover, because the size of the current is limited by the induced e.m.f. and the resistance of the ring. If the same experiment is carried out using a superconducting ring, then it can be made to float. Alternatively, a small magnet will float above a superconducting dish. The currents induced in superconductors are always large enough to completely cancel the flux crossing their surface. (This is known as the **Meissner effect**.)



The eddy currents in a superconductor are so strong that the repulsion between them and the currents in a magnet can cause the magnet to float above the surface of the superconductor.

Currents can be induced in ordinary pieces of metal. Although the e.m.f.s are generally not very large, the **eddy currents** follow low resistance paths through the metallic structure and can be quite large. Lenz's law implies that they act to oppose the motion that produces them. They can act as a very efficient braking mechanism as the kinetic energy is dissipated as heat. Their main drawback is that as the motion slows down so the eddy currents become smaller, and the braking effect is reduced.

Whirlpools and eddies

The currents induced in metals follow paths that are small loops inside the body of the metal. As these remind people of the small whirlpools formed in streams (eddies) they are referred to as eddy currents.

PRACTICE

1 A straight wire is moving horizontally through a vertical magnetic field at a constant speed v. Sketch a diagram of the position of the wire at two moments A and B which are 1 s apart. Consider the rectangle swept out by the wire as it moves from A to B. What happens to the flux linked to this rectangle as the wire moves from A to B? Use Lenz's law to work out the direction of the current induced in the wire. (Consider the magnetic field of the induced current.) Does this agree with the direction suggested by Faraday's law?

2 A soft iron rod is placed through the axis of a solenoid and an aluminium ring is threaded down the rod to rest on the solenoid. A constant d.c. current is now turned on in the solenoid. What happens to the ring? Explain your answer. The current is now turned off. State and explain what now happens to the ring. A ring is now cut so that it forms a 'C' shape when viewed from above, and the experiment is repeated. This time nothing happens. Explain.

3 Describe a simple experiment that you could use to demonstrate Lenz's law to a non-scientific audience.

5.24 EXAMPLES OF INDUCTION

OBJECTIVES

- e.m.f. in a rotating coil
- mutual induction

Helpful hint

In the next few sections:

N is the total number of turns in a coil

n is the number of turns per unit length

Helpful hint

Differentiating sin and cos functions is covered in the Mathematics toolbox. Remember, in the equation for e.m.f. the angle ωt is in radians, and your calculator should be set accordingly.

The electrical tether being developed by NASA is not a practical method for generating electricity on Earth – the speeds involved are too great, for one thing. However, the same basic effect – the magnetic induction of a current – can be used by rotating a coil in a magnetic field.

A rotating coil in a magnetic field

The diagram below shows a flat rectangular coil of N turns being rotated at a constant angular velocity ω in a magnetic field. At the instant shown, the coil is at an angle θ to the magnetic field direction. The flux linked to the coil is given by the component of the field strength passing through the coil at 90°, that is

$$N\phi = \text{area of coil} \times \text{number of turns} \times B\sin\theta$$

Assuming that the coil was horizontal when we started timing the rotation:

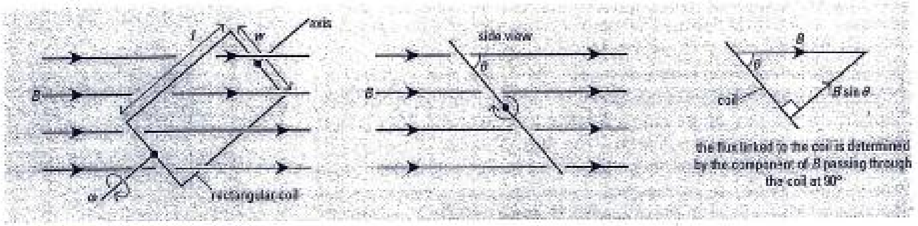
$$\theta = \text{angular velocity} \times \text{time} = \omega t$$

$$\therefore \quad N\phi = BAN\sin\omega t$$

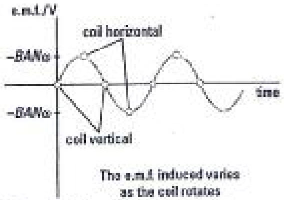
The flux linked to the coil alters as the coil rotates, and Faraday's law can be used to calculate the e.m.f. induced:

$$\varepsilon = -\frac{d(N\phi)}{dt} = -\frac{d(BAN\sin\omega t)}{dt}$$

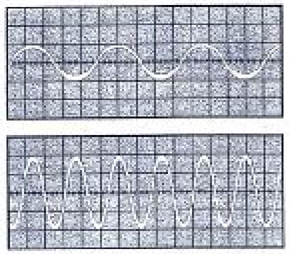
$$\therefore \quad \varepsilon = -BAN\omega\cos\omega t$$



A rotating coil in a magnetic field.



The e.m.f. in a rotating coil as a function of time.



Oscilloscope traces of the e.m.f. in a rotating coil.

There are several important points relating to this result:

- When $t = 0$ or π/ω, the coil is *horizontal* and the *flux linked* is therefore *zero*.
- When the coil is *vertical* the *flux linked* is a *maximum*, $\phi = BNA$.
- When the coil is *horizontal*, the *induced e.m.f.* is a *maximum*, $\varepsilon = -BNA\omega$.
- When the coil is *vertical* the *induced e.m.f.* is *zero*.

This seems contradictory, but remember that the e.m.f. is determined by the rate at which the flux *changes*. As the coil passes through the horizontal position the flux is changing rapidly; as the coil is moving through the vertical position the flux is hardly changing.

- Every time the coil passes through the vertical the current reverses direction.
- The size of the e.m.f. is proportional to the rate at which the coil is rotating.

The diagram on the left shows an oscilloscope trace recording the e.m.f. induced in a rotating coil. In the second trace the coil speed has been doubled. Notice that the frequency at which the e.m.f. changes direction has increased as well as the size of the e.m.f.

This technique can be used for generating electric current. In practice, turbines work by keeping a fixed coil and rotating the magnetic field generators.

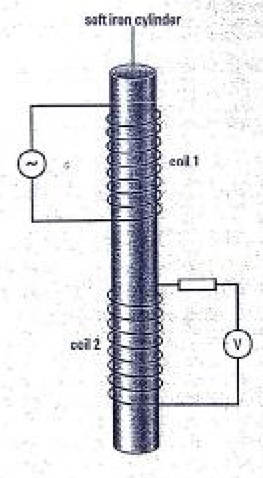
Magnetically linked coils

The diagram on the right shows two coils wound round the same piece of soft iron. This arrangement ensures that the coils are magnetically linked to each other. Passing an alternating current through one of the coils induces an alternating e.m.f. in the other. This effect, known as **mutual induction**, is used in the design of transformers that can step voltages up or down.

The alternating current in coil 1 produces a varying magnetic field that magnetizes the soft iron in the same varying pattern. This ensures that there is a changing magnetic field passing through the centre of coil 2. A high-resistance voltmeter connected to this coil records the changing e.m.f. that is electrically induced. This e.m.f. is proportional to the rate at which the current is varying in coil 1.

$$\varepsilon_2 \propto -\frac{d(I_1)}{dt} = -M\frac{d(I_1)}{dt}$$

where I_1 is the size of the current in coil 1 at any time t and the constant of proportionality, M, is the **mutual inductance** of the two coils measured in the henry (H).



Mutual induction in two coils wound round the same piece of soft iron.

Maths box: deriving the equation for mutual inductance of two coils

If I_1 is the size of the current in coil 1 at any time t then the magnetic field in coil 1 is

$B_1 = \mu_0 n_1 I_1$ (n_1 is the number of turns per metre in coil 1)

The field passing through coil 2 is

$B_2 = \mu_r \mu_0 n_1 I_1$

where μ_r is the **relative magnetic permeability** of the soft iron. Consequently, the flux linked to the second coil is

$\phi_2 = A_2 N_2 B_2$ (N_2 is the number of turns in coil 2)

$= A_2 N_2 \mu_r \mu_0 n_1 I_1$

The e.m.f. induced by this changing flux is given by Faraday's law:

$$\varepsilon = -\frac{d(A_2 N_2 \mu_r \mu_0 n_1 I_1)}{dt}$$

Everything in the bracket is constant apart from I_1, so

$$\varepsilon = -A_2 N_2 \mu_r \mu_0 n_1 \frac{d(I_1)}{dt} = -M\frac{d(I_1)}{dt} \therefore M = A_2 N_2 \mu_r \mu_0 n_1$$

Mutual inductance, M, is a constant and depends on the number of turns, area of the coils, etc. NB This equation applies only to the configuration shown in the diagram.

PRACTICE

1 What is the angular velocity equivalent to 30 r.p.m. (revolutions per minute)? What is the angular velocity of the minute hand on a clock?

2 What is the maximum e.m.f. that can be obtained from a rotating coil of 100 turns in a magnetic field of 0.2 T, if the area of the coil is 2.5 cm^2 and it is rotating at 20 r.p.m.?

3 Sketch a graph showing how the flux cut by the rotating coil in the diagram on the previous page varies with time for 2 complete rotations. On the same set of axes, sketch a graph showing how the induced e.m.f. varies with time. Be sure to include relevant information on the vertical axes of both graphs.

4 In the situation shown in the diagram above, the current in coil 1 is changing at 0.5 A s^{-1}. The e.m.f. in coil 2 is 0.1 V. What is the mutual inductance of the two coils? The situation is now reversed – the changing current is in coil 2 and coil 1 is connected to the ammeter. What rate of change of current in coil 2 is required to produce an e.m.f. of 0.3 V in coil 1? (Assume that the two coils are of the same length and cross-sectional area.)

5 What is the henry in base units?