

EXERCISE 9E

- Evaluate using factor form: **a** C_1^8 **b** C_2^8 **c** C_3^8 **d** C_6^8 **e** C_8^8 .
Check each answer using your calculator.
- In question 1 you probably noticed that $C_2^8 = C_6^8$.
In general, $C_r^n = C_{n-r}^n$. Prove that this statement is true. (**Hint:** Use factorial form.)
- List the different teams of 3 that can be chosen from a squad of 5 (named A, B, C, D and E). Check that the formula for C_r^n gives the total number of teams.
- How many different teams of 11 can be chosen from a squad of 17?
- Candidates for an examination are required to do 5 questions out of 9. In how many ways can this be done? If question 1 was compulsory, how many selections would be possible?
- How many different committees of 3 can be selected from 13?
How many of these committees consist of the president and 2 others?

Example 13

A committee of 4 is chosen from 7 men and 6 women. How many different committees can be chosen if:

- there are no restrictions
- there must be 2 of each sex
- at least one of each sex is needed?

a For no restrictions there are $7 + 6 = 13$ people up for selection and we want any 4 of them. \therefore total number $= C_4^{13} = 715$.

b The 2 men can be chosen in C_2^7 ways and the 2 women can be chosen in C_2^6 ways.
 \therefore total number $= C_2^7 \times C_2^6 = 315$.

c Total number
= number with (3 M and 1 W) or (2 M and 2 W) or (1 M and 3 W)
 $= C_3^7 \times C_1^6 + C_2^7 \times C_2^6 + C_1^7 \times C_3^6$
 $= 665$

Alternatively, total number $= C_4^{13} - C_4^7 \times C_0^6 - C_0^7 \times C_4^6$. Why?

- How many different teams of 5 can be selected from a squad of 12?
How many of these teams contain:
a the captain and vice-captain **b** exactly one of the captain or the vice-captain?
- A team of 9 is selected from a squad of 15 of which 3 are *certainties*, i.e., must be included, and another must be excluded because of injury. In how many ways can this be done?

- 9 In how many ways can 4 people be selected from 10 if:
- a one person is always in the selection
 - b 2 are excluded from every selection
 - c 1 is always included and 2 are always excluded?
- 10 A committee of 5 is chosen from 10 men and 6 women. Determine the number of ways of selecting the committee if:
- a there are no restrictions
 - b it is to contain 3 men and 2 women
 - c it is to contain all men
 - d it is to contain at least 3 men
 - e it is to contain at least one of each sex.
- 11 A committee of 5 is chosen from 6 doctors, 3 dentists and 7 others. Determine the number of ways of selecting the committee if it is to contain:
- a 2 doctors and 1 dentist
 - b 2 doctors
 - c at least one of the two professions.
- 12 How many diagonals has a 20-sided convex polygon?
- 13 There are 12 distinct points A, B, C, D, ..., L, on a circle.
- a How many lines
 - i are determined by the points
 - ii pass through B?
 - b How many triangles
 - i are determined by the points
 - ii have one vertex B?
- 14 How many 4-digit numbers can be constructed where the digits are in ascending order from left to right? **Note:** You cannot start with 0. Why?
- 15 a Give an example which demonstrates that:
- $$C_0^5 \times C_4^6 + C_1^5 \times C_3^6 + C_2^5 \times C_2^6 + C_3^5 \times C_1^6 + C_4^5 \times C_0^6 = C_4^{11}.$$
- b Copy and complete:
- $$C_0^m \times C_r^n + C_1^m \times C_{r-1}^n + C_2^m \times C_{r-2}^n + \dots + C_{r-1}^m \times C_1^n + C_r^m \times C_0^n = \dots$$
- 16 In how many ways can 12 people be divided into:
- a two equal groups
 - b three equal groups?