Circle = circumference is a set of points X in the plane which have the same distance **r** from the fixed point **S**.

|SX| = r = X – S

If X[x, y] and S[m, n], then the analytical expression of a circumference with the centre S and radius r is: $\mathbf{(x - m)^2 + (y - n)^2 = r^2}$

This equation is called the *centre form of the circumference equation*. If we raise the brackets to the powers, we get the *general form of the circle equation*: $x^2 + y^2 - 2mx - 2ny + m^2 + n^2 - r^2 = 0$, what is in fact: $x^2 + y^2 + ax + by + c = 0$.

Basic terminology:

A **chord** is an abscissa that joins two points on the circumference of a circle.

A **tangent** to a circle is a straight line that touches the circle at one point only.

A **diameter** is a chord that passes through the centre.

Circumference is the distance measured around the curved edge of a circle.

A **radius** is any abscissa from the centre to the circumference.

The curved part of the circle between points A and B is known as an **arc**. There are two arcs AB: The larger is the **major arc**, the smaller is the **minor arc**.

The chord AB divides the circle into two **segments**, the major segment and the minor segment.

The **perpendicular bisector** of any chord passes through the centre of its circle (i.e. it is a line that is perpendicular to the chord and passes through its centre).

Any straight line that cuts across a circle at two points is called a **secant**. (A tangent is a special case of a secant, as the two cuts become a single point at contact. These two cuts are said to become coincident.)

TANGENTS TO A CIRCLE

The angle between a radius, drawn to the point of contact of a tangent, and the tangent itself is a right angle.

From any point P, outside the circle, two tangents can be drawn; the distance from P to the point of contact being the same for each tangent.

If point X lies:

a) On the circumference, then $\mathbf{(x - m)^2 + (y - n)^2 = r^2}$
b) Inside the circle, then $\mathbf{(x - m)^2 + (y - n)^2 < r^2}$

c) Outside the circle, then $\mathbf{(x - m)^2 + (y - n)^2 > r^2}$

Mutual position of a circle and other linear shapes

Ex: Circle k has a centre S[2, 3] and r = 5. You're given a point A[-3, -4] and B[1, 6]. Find the intersection of AB and the circle k.

We need to write the centre form of circle equation: $(x - 2)^2 + (y - 3)^2 = 25$, and the vector equations of a line AB: $x = -3 + 4t$, $y = -4 + 10t$.

We substitute for x, and y into the quadratic equation and we get a value of parameter t.

If AB is an abscissa then $t \in \langle 0,1 \rangle$.

Tangent to a circle

- Contains one point of a circle only
- The centre of the circle has a distance of r from it
- Is perpendicular to that radius of a circle, which contains the point of contact

The equation $(x_0 - m)(x - m) + (y_0 - n)(y - n) = r^2$ is the analytical expression of a tangent to a circle $(x - m)^2 + (y - n)^2 = r^2$ at a point $T[x_0, y_0]$.