Analytical geometry of linear shapes

YEAR 3

Mgr. Oľga Štureková

Basic terms

A *line* is given by 2 different points or one point and a vector.

The *line segment (abscissa, pl. abscissae)* is the straight line joining some point A to some other point B.

A *ray* is a line with a start point but no end point (it goes to infinity).

A *plane* is a two-dimensional surface. It is given by three

noncollinear (do not lie on the same line) points or by a point and two noncollinear vectors.

Space is a three-dimensional or **3D** because there are three dimensions: *width, depth* and *height.*

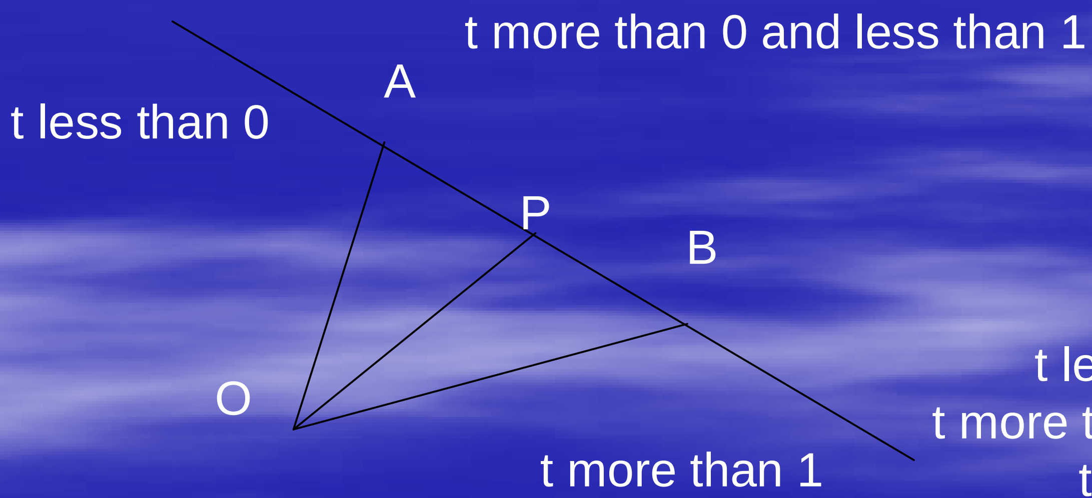
A direction vector shows the direction of a line.

A normal vector is a perpendicular vector to the direction of the line.

Vector equation of a line

Let ***OA*** and ***OB*** be the position vectors of two points A and B with respect to an origin O. Let ***OP*** be the position vector of a point P on the line AB.

OP = OA+AP = OA + t AB, where t is a scalar (parameter).



t less than 0 - a ray

t more than 0 - an opposite ray

t is 0 - a point A

t is 1 - a point B

t more than 0 and less than 1 – a segment

Vector equation of a line in a plane

The line is t multiple of the line AB

$$\mathbf{AP} = t\,(\mathbf{B} - \mathbf{A})$$

$$\mathbf{P} - \mathbf{A} = t\,(\mathbf{B} - \mathbf{A}) \qquad /+A$$

$$\mathbf{P} = \mathbf{A} + t\,(\mathbf{B} - \mathbf{A})$$

Any point of the line can be found as one point of the line plus t times direction vector.

$$[x, y] = [x_A, y_A] + t\,(x_B - x_A; y_A - y_B)$$

is a vector equation of a line AB.

The parametric equations of a line in a plane

Given two points $A=[x_1, y_1]$ and $B=[x_2, y_2]$

and vector *AB*= (x_2-x_1, y_2-y_1),

$$P = A + t\,\mathbf{AB}$$

$$[x, y] = [x_A, y_A] + t\,(x_B - x_A; y_A - y_B)$$

(Distinguish x and y coordinates)

The parametric equations of a line in a plane are:

$$x = x_1 + t\,(x_2-x_1),$$

$$y = y_1 + t\,(y_2-y_1), \text{ t belongs to R}$$

The midpoint formula

Let $A=[x_1, y_1]$ and $B=[x_2, y_2]$ be two points. The midpoint between them has coordinates

$$S = [(x_A+x_B)/2; (y_A+y_B)/2]$$

Exercise 1

Find equations of the line that passes through A = [-2,1] and B= [1,3] and find the coordinates of the midpoint.

(a) Direction Vector: B – A = ***(3,2)***.

(b) Vector Equation:

[x, y] = [-2,1] + t (3,2)

(c) Parametric Equations:

x = -2 + 3t,
y = 1 + 2t, t belongs to R

(d) Midpoint: [(1-2)/2; (1+3)/2]
[-0,5; 2]

Exercise 2

Find out if a point C=[1;2] lies on the line that passes through A = [-2,1] and B= [1,3].

Parametric Equations are ***x = -2 + 3t,***
y = 1 + 2t, t belongs to R

Put coordinates of C instead of x and y.
1 = -2 + 3t, so $t_1 = 1$
2 = 1 + 2t, so $t_2 = 0{,}5$

t_1 is different from t_2 so C doesn't lie on this line.

Vector equation of a line in space

The line is t multiple of the line AB

$$AP = t\,(B - A)$$

$$P - A = t\,(B - A) \qquad /+A$$

$$P = A + t\,(B - A)$$

Any point of the line can be found as one point of the line plus t times direction vector.

$$[x, y, z] = [x_A, y_A, z_A] + t\,(x_B - x_A;\ y_A - y_B;\ z_B - z_A)$$

is a vector equation of a line AB in space.

The parametric equations of a line in space

Given two points $A=[x_1, y_{1\,,}\ z_{\,1}]$ and $B=[x_2, y_2\ z_{\,2}]$
and vector *AB*= $(x_2-x_1, y_2-y_{1,}\ z_{\,2}-z_{\,1})$,

$$P = A + t\,\mathbf{AB}$$

$$[x, y, z] = [x_A, y_A, z_A] + t\,(x_B - x_A; y_A - y_B; z_B - z_A)$$

(Distinguish x and y coordinates)

The parametric equations of a line in a plane are:

$$x = x_1 + t\,(x_2-x_1),$$
$$y = y_1 + t\,(y_2-y_1),$$
$$z = z_1 + t\,(z_2-z_1),\ t \text{ belongs to } R$$

The Cartesian equation of a line (general form)

$a x + b y + c = 0$,

where a, b, and c are real numbers, and both a and b are not zero

a and b are components of a normal vector of a line

x and y are coordinates of a point

c is a y-axis intercept

The Cartesian equation of a line (general form)

The Cartesian equation of the line can be found from the parametric form.
The vector equation of the line is

$$[x, y] = [3,1] + t\,(1,-2)$$

The parametric equations of the line are
$x = 3 + t$,
$y = 1 – 2t$, t belongs to R

Eliminate t term (use addition or substitution method).

$x = 3 + t$, so $t = x – 3$
$y = 1 – 2t$,
$y = 1 – 2(x – 3)$
$y = 1 - 2x + 6$

Put everything to one side.

$y + 2x – 7 = 0$

The Cartesian equation of the line is **$2x + y – 7 = 0$**

The Cartesian equation of a line (general form) in a plane

The other way how to find the Cartesian equation of the line is to find a normal vector **n** (a normal vector of the line) that is perpendicular to **vector**

v (1,-2) **n** . **v** = 0

if **a** = (a_1,a_2) then **n** is $(a_2, - a_1)$

change x and y component, then change the sign before one component

v = (1,-2) then **n** = (-2, - 1) or (2, 1)

point A lies on the line so

$$-2(3) – 1(1) + c = 0$$
$$- 6 – 1 = - c$$
$$c = 7$$

The Cartesian equation of the line is -2x - y + 7 = 0
It is -1 multiple of 2x + y – 6 = 0
They express the same line.

The Cartesian equation of a line (general form) in space

There is no Cartesian equation of a line in space.

It can be written just for the line in a plane.