

Solving Linear Equations

A linear equation is a first degree expression in the forms $x + a = b$, $ax + b = c$, or $ax + b = cx + d$, where a , b , c and d are any real numbers and x is the unknown variable. The purpose of solving a linear equation is to find the value(s) for the variable(s) that will make the equation true, or in other words, make the right side equal the left side. Examples of linear equations are:

Ex. $2x + 5 = 0$

Ex. $20 + x = 4 - 15x$

Ex. $-70 = x - 48$

NOTE: All variables in a first-order expression must have an exponent of 1.

Steps in solving any linear equation:

The goal is to move all variables to one side of the equal sign and all constants to the right side of the equal sign. **Variable = Constant**

The rules of operations must always be used to solve equations.

- 1) If the equation has parentheses, using the distributive property will eliminate the parentheses.
- 2) Combining like terms (constants with constants and variables with variables) on each side of the equal sign simplifies the equation.
- 3) Performing opposite operations to eliminate constants on the left and variables on the right will result in having an equation in the form **Variable = Constant**

NOTE: The opposite operation to division is multiplication and vice versa.

The opposite operation to addition is subtraction and vice versa.

- 4) Check the solution by substituting it into the original equation.

EXAMPLE

Solve the following equation: $5(3x-1) + 2 = 12x + 6$

$5(3x-1) + 2 = 12x + 6$ distribute the 5 by multiplying it with the terms inside the parenthesis.

$15x - 5 + 2 = 12x + 6$ combine constant like terms ($-5+2 = -3$)

$15x - 3 = 12x + 6$ combine variables on the right side by subtracting $12x$ from both sides

$\frac{-12x}{3x} \quad \frac{-12x}{0}$ **NOTE:** Subtracting $12x$ from both sides will not change the equation.

$3x - 3 = 6$ combine constants on the left side by adding 3 to both sides

$\frac{+3}{0} \quad \frac{+3}{9}$

$\frac{3x}{3} = \frac{9}{3}$ divide both sides by 3 to eliminate the coefficient in front of the x.

$x = 3$ The equation satisfies **Variable = Constant**. Therefore the solution is $x=3$.

Check your solution by substituting every x in the equation with a 3.

$$5(3x-1) + 2 = 12x + 6$$

$$5[3(3) - 1] + 2 = 12(3) + 6$$

$$5(9 - 1) + 2 = 36 + 6$$

$$5(8) + 2 = 42$$

$$40 + 2 = 42$$

$$42 = 42$$

NOTE: Remember PEMDAS

(Parenthesis, Exponents, Multiplication, Division, Addition, Subtraction)

CROSS MULTIPLICATION

There are some problems in the form of a fraction that are just another form of a linear equation. Such problems are easier to solve by using the cross multiplication method.

Given $\frac{a}{b} = \frac{c}{d}$, we can convert it to $a \cdot d = b \cdot c$

Steps in using the cross multiplication:

- 1) Multiply the numerator on the left side, a , of the equation with the denominator of the right side, d , of the equation and the multiply the numerator of the right equation, c , with the denominator of the left equation, b . $a \cdot d = b \cdot c$
- 2) Isolate the variable in **Variable = Constant** form by performing opposite operations.

EXAMPLE

Solve the equation: $\frac{2x+1}{7} = \frac{x-3}{5}$

$$\frac{2x+1}{7} \begin{array}{c} \swarrow = \searrow \\ \nwarrow \quad \nearrow \end{array} \frac{x-3}{5}$$

apply the cross multiplication method

$$5(2x+1) = 7(x-3)$$

distribute the 5 on the left side and the 7 on the right side

$$10x + 5 = 7x - 21$$
$$\frac{-5}{0} \quad \frac{-5}{-26}$$

combine like constant terms by subtracting 5 on both sides

$$10x = 7x - 26$$

combine like variable terms by subtracting $7x$ from both sides

$$\frac{-7x}{3x} \quad \frac{-7x}{0}$$

$$\frac{3x}{3} = \frac{-26}{3}$$

divide both sides by 3 to isolate the variable x

$$x = \frac{-26}{3}$$

the solution is $x = -26/3$