Solving Quadratic Equations by Factoring

A handout from Learning Assistance at Trident Technical College

Equations that can be written in the form $Ax^2 + Bx + C = 0$

where a, b, and c are integers and a>0 are called **quadratic equations**. This form is called the **standard form**.

The easiest way to solve quadratic equations is to factor (if possible) the polynomial on the left side of the equation above. We then use the **Zero-Product Theorem**.

> Zero-Product Theorem for any real numbers a and b, if ab = 0, then a = 0 or b = 0.

Quadratic Equations:

1. $x^2 - 3x - 4 = 0$ — This quadratic equation is **in standard form**.
2. $A^2 = 6A - 9$ — This quadratic equation is **not in standard form**.
3. $(x - 7)(x + 6) = -22$ — This is a quadratic equation **not in standard form**.
4. $(x + 8)(x - 5) = 0$ — By the **Zero-Product Theorem**, the quadratic equation can be solved.
5. $x(x - 14) = 0$ — This quadratic equation is **in factored form and equal to zero**; it can be solved **by using the Zero-Product Theorem**.

> To solve quadratic equations by factoring:
>
> 1. Put the equation into standard form.
> 2. Factor the polynomial.
> 3. Set each factor equal to zero.
> 4. Solve each of the first degree equations.

EXAMPLE 1: $2x^2 - 9x - 35 = 0$

Solution: This is already in standard form, so we start by factoring.

Factor: $(2x + 5)(x - 7) = 0$

Set each factor = 0: $(2x + 5) = 0$ or $(x - 7) = 0$

Solve each equation: $2x + 5 = 0$ or $x - 7 = 0$

$2x = -5$; $x = 7$

$\frac{2x}{2} = \frac{-5}{2}$

The solution set is: $\left\{-\frac{5}{2}, 7\right\}$

EXAMPLE 2: $a^2 = 6a - 9$

Solution: We first put this into standard form.

The equation is not in standard form	$a^2 = 6a - 9$
Put into standard form	$a^2 - 6a + 9 = 0$
Factor	$(a - 3)(a - 3) = 0$
Set each factor = 0	$a - 3 = 0$ or $a - 3 = 0$
Solve each equation	$a - 3 = 0$ $\quad$ $a - 3 = 0$ $a = 3$ $\quad$ $a = 3$

The solution set is: {3,3}

EXAMPLE 3: $(t - 7)(t + 6) = -22$

Solution: Multiple this out and put into standard form.

FOIL	$t^2 - t - 42 + 22 = 0$
Standard form	$t^2 - t - 20 = 0$
Factor	$(t - 5)(t + 4) = 0$
Set each factor = 0	$t - 5 = 0$ or $t + 4 = 0$
Solve each equation	$t = 5$ $\quad$ $t = -4$

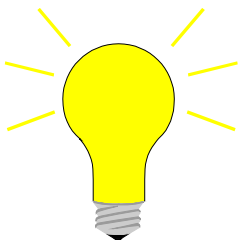
The solution set is: {5,–4}

EXAMPLE 4: Solve $x(x - 14) = 0$

Solution: The equation is in factored form and equal to zero. It can be solved by using the **Zero-Product Theorem**.

Factor	$x(x - 14) = 0$
Set each factor = 0	$x = 0$ or $x - 14 = 0$ $x = 14$

The solution set is: {0,14}



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