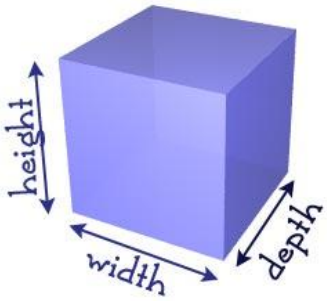
Solid Geometry



Solid Geometry is the geometry of three-dimensional space, the kind of space we live in.
Three Dimensions

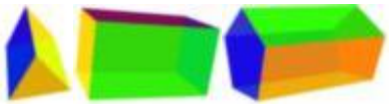
It is called **three-dimensional** or **3D** because there are three dimensions: *width, depth* and *height.*

There are two main types of solids, "Polyhedra", and "Non-Polyhedra":

Polyhedra

A polyhedron is a solid made of flat surfaces. A **polyhedron** is a solid with flat faces (from Greek poly- meaning "many" and -edron meaning "face").
Each flat surface (or "face") is a polygon. So, to be a polyhedron there should be **no curved surfaces**. Examples of Polyhedra:


Platonic Solids


Prisms


Pyramids

Non-Polyhedra

Some solids have curved surfaces or a mix of curved and flat surfaces (so they aren't polyhedra). Examples:


Sphere


Torus

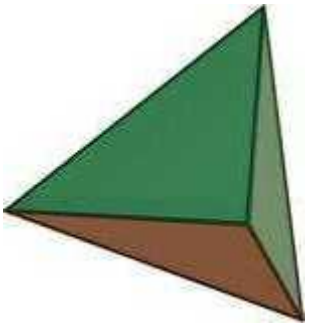

Cylinder


Cone

Properties
Solids have *properties* (special things about them), such as:
Volume (think of how much water it could hold)
Surface area (think of the area you would have to paint)

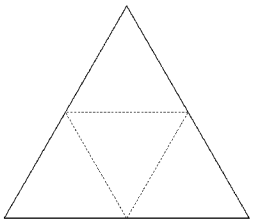
Platonic Solids

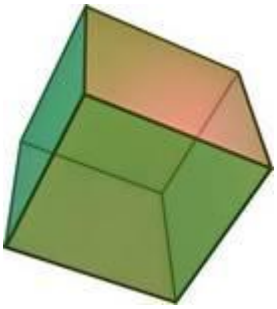
There are five platonic solids. For each solid there are two printable nets (with and without tabs). These nets can be printed onto a piece of card. You can then make your own platonic solids by cutting them out and taping the edges together.



Tetrahedron
4 Faces
4 Vertices
6 Edges

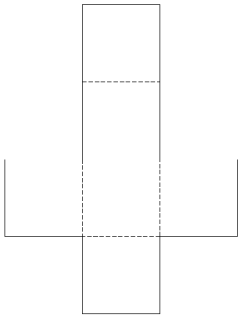
Tetrahedron Net

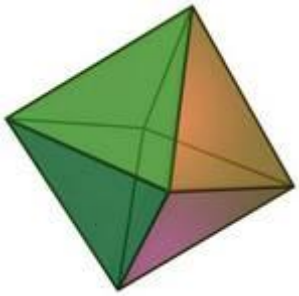




Cube
6 Faces
8 Vertices
12 Edges

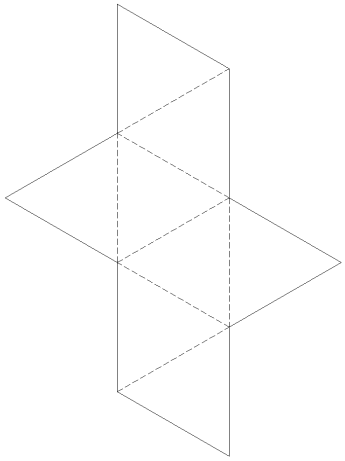
Cube Net





Octahedron
8 Faces
6 Vertices
12 Edges

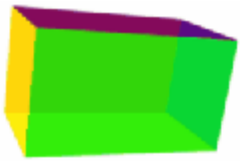
Octahedron Net



Prisms

A prism has the same cross section all along its length !
These are all Prisms:

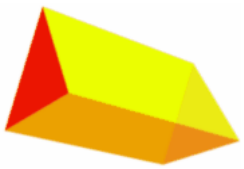
Square Prism:



Cross-Section:

Cube:

Triangular Prism:

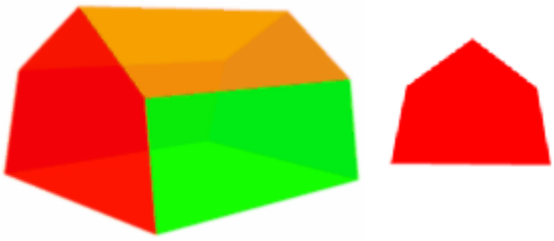


Cross-Section

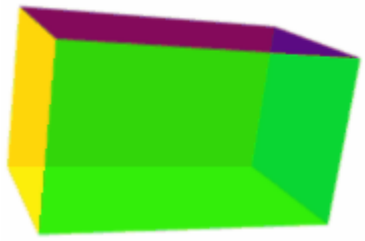
:

Pentagonal Prism: Cross-Section:

Regular and Irregular Prisms
Here is an example of an **Irregular Prism**:
Irregular Pentagonal Prism: Cross-Section:



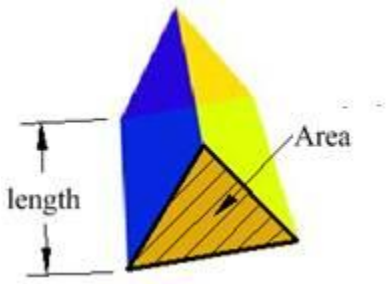
Cuboids and Rectangular Prisms
A **cuboid** is a box-shaped object. It has six flat sides and all angles are right angles.
And all of its faces are rectangles.
It is also a prism because it has the same cross-section along a length. In fact it is a **rectangular prism**.



If at least two of the lengths are equal then it can also be called a **square prism**.
(Note that this doesn't stop it from also being called a rectangular prism if you want!)

So a cube is just a special case of a square prism, and a square prism is just a special case of a rectangular prism.
And they are all cuboids.

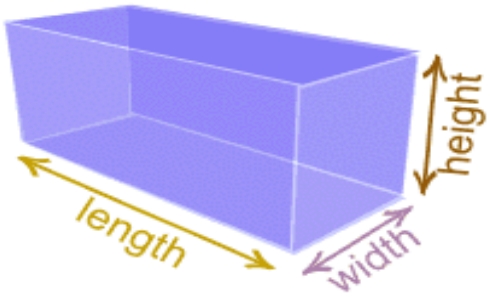
Volume of a Prism



The Volume of a prism is simply the area of one end times the length of the prism

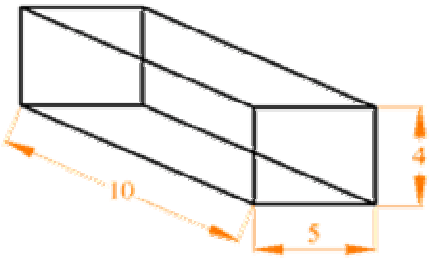
Volume = Area × Length
Example: What is the volume of a prism whose ends are 25 in^2 and which is 12 in long:
Answer: Volume = 25 in^2 × 12 in = 300 in^3

Volume and Surface Area



The volume of a cuboid is simply:
Volume = width × depth × height
V = wdh
And the surface area is:
A = 2wd + 2dh + 2hw

Example
Find the volume and surface area of this cuboid.



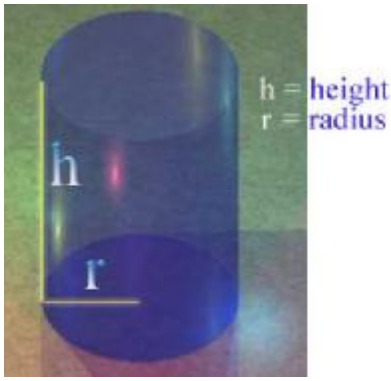
V = 4×5×10 = 200

A = 2×4×5 + 2×5×10 + 2×10×4
= 40+100+80 = 220

Units of volume include:
Metric: cubic centimeters (cm^3), cubic meters (m^3), liters

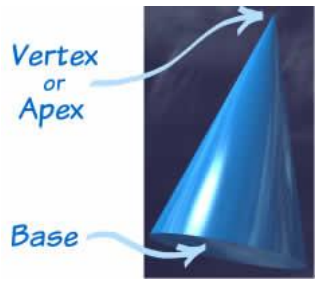
Cylinder
It has a flat base and a flat top. The base is the same as the top, and also in-between It has one curved side Because it has a curved surface it is not a polyhedron.
Surface Area = $2 \times \boldsymbol{\pi} \times r \times (r + h)$
Surface Area of One End = $\boldsymbol{\pi} \times r^2$
Surface Area of Side = $2 \times \boldsymbol{\pi} \times r \times h$
Volume = $\boldsymbol{\pi} \times r^2 \times h$

Volume of a Cylinder
Just multiply the area of the circle by the height of the cylinder:
Area of the circle: $\pi \times r^2$
Height: h
Volume = Area × Height = $\pi \times r^2 \times h$
An object shaped like a cylinder is said to be **cylindrical**

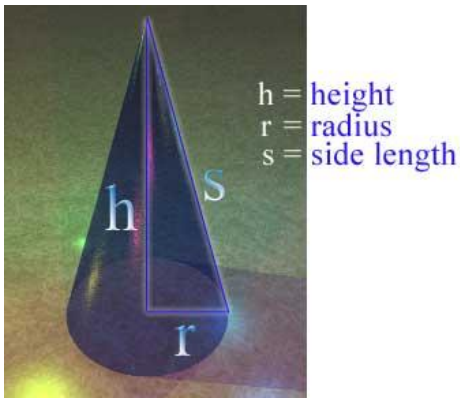


Volume of a Cylinder
Just multiply the area of the circle by the height of the cylinder:
Area of the circle: $\pi \times r^2$
Height: h
Volume = Area × Height = $\pi \times r^2 \times h$

Cone

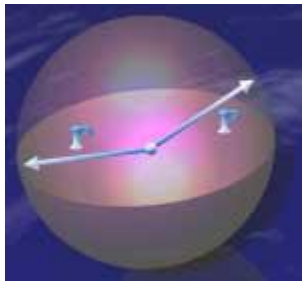


A Cone is a Rotated Triangle
A cone is made by rotating a triangle!
The side it rotates around is the **axis** of the cone. It has a flat base It has one curved side. Because it has a curved surface it is not a polyhedron. The pointy end of a cone is called the **vertex** or **apex.** The flat part is the **base**. An object shaped like a cone is said to be **conical.**



Surface Area of *Base* = $\pi \times r^2$
Surface Area of *Side* = $\pi \times r \times s$
Surface Area of *Side* = $\pi \times r \times \sqrt{(r^2+h^2)}$
Volume = $\pi \times r^2 \times (h/3)$

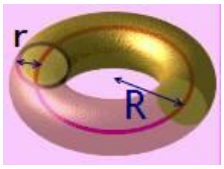
Sphere

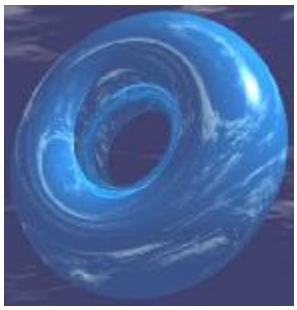


It is perfectly symmetrical. It has no edges or vertices. It is **not** a polyhedron. All points on the surface are the same distance from the center.
Surface Area = $4 \times \pi \times r^2$
Volume = $(4/3) \times \pi \times r^3$

The Earth
The Planet Earth, our home, is *nearly* a sphere, except that it is squashed a little at the poles It is a **spheroid**, which means it just misses out on being a sphere because it isn't perfect in one direction (in the Earth's case: North-South).

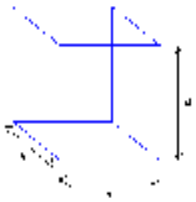
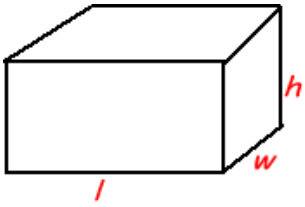
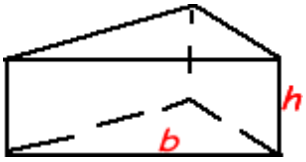
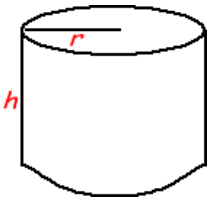
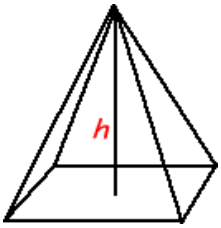
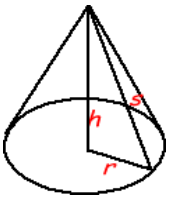
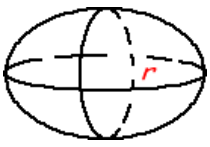
Torus





It can be made by revolving a small circle along a line made by another circle. It has no edges or vertices. It is **not** a polyhedron. Surface Area = $4 \times \pi^2 \times R \times r$
Volume = $2 \times \pi^2 \times R \times r^2$
Note: Area and volume formulas only work when the torus has a hole! If you have more than one torus they are called **tori.**

Volume and surface area

	**Cube** Volume= $a^3 = a \times a \times a$ $V = a^3$ Surface **=** a^2 **+** a^2 **+** a^2 **+** a^2 **+** a^2 **+** a^2 $S = 6 \times a^2$
	**Rectangular Solid** Volume **=** Length **X** Width **X** Height **V = lwh** Surface **= 2lw + 2lh + 2wh**
	**Prisms** Volume **=** Base **X** Height **V=bh** Surface **= 2b + Ph** *(b is the area of the base P is the perimeter of the base)*
	**Cylinder** Volume **=** πr^2 **x** height **V =** πr^2 **h** Surface **=** 2π radius **x** height **S =** $2\pi rh + 2\pi r^2$
	**Pyramid** **V = 1/3 bh** b is the area of the base Surface Area: Add the area of the base to the sum of the areas of all of the triangular faces. The areas of the triangular faces will have different formulas for different shaped bases**.**
	**Cones** Volume **=** $\frac{1}{3}\pi r^2$ **x** height **V=** $\frac{1}{3}\pi r^2 h$ **S =** $\pi r^2 + \pi rs = \pi r^2 + \pi r\sqrt{r^2 + h^2}$
	**Sphere** Volume **=** $\frac{4}{3}\pi r^3$ **V =** $\frac{4}{3}\pi r^3$ Surface **=** $4\pi r^2$ **S =** $4\pi r^2$