

SOLVING RATIONAL EQUATIONS EXAMPLES

1. Recall that you can solve equations containing fractions by using the least common denominator of all the fractions in the equation. Multiplying each side of the equation by the common denominator eliminates the fractions. This method can also be used with rational equations. **Rational equations** are equations containing rational expressions.

2. **Example:** solve $\rightarrow \frac{x-4}{4} + \frac{x}{3} = 6$.

$$\begin{aligned}\frac{x-4}{4} + \frac{x}{3} &= 6. \\ 12\left(\frac{x-4}{4} + \frac{x}{3}\right) &= 12(6) \\ 3(x-4) + 4(x) &= 72 \\ 3x - 12 + 4x &= 72 \\ 7x &= 84 \\ x &= 12\end{aligned}$$

The LCD of the fraction is 12.

Multiply each side of the equation by 12. The fractions are eliminated.

Emphasize that each term must be multiplied by the LCD in order to have a balanced equation. A common mistake is to multiply only those terms that are expressed in fractions.

Check $\rightarrow \frac{x-4}{4} + \frac{x}{3} = 6 \rightarrow \frac{12-4}{4} + \frac{12}{3} = 6 \rightarrow 2 + 4 = 6 \rightarrow 6 = 6$

3. **Example:** solve $\rightarrow \frac{3}{2x} - \frac{2x}{x+1} = -2$

Note that $x \neq -1$ and $x \neq 0$. The LCD of the fractions is $2x(x+1)$

$$\frac{3}{2x} - \frac{2x}{x+1} = -2 \rightarrow$$

$$2x(x+1)\left(\frac{3}{2x} - \frac{2x}{x+1}\right) = 2x(x+1)(-2)$$

$$3(x+1) - 2x(2x) = -4x^2 - 4x$$

$$3x + 3 - 4x^2 = -4x^2 - 4x$$

$$7x = -3$$

$$x = -\frac{3}{7}$$

Multiply each side of the equation by $2x(x+1)$.

Check $\rightarrow \frac{3}{2(-\frac{3}{7})} - \frac{2(-\frac{3}{7})}{(-\frac{3}{7}+1)} = -2$

$$\frac{3}{-\frac{6}{7}} - \frac{-\frac{6}{7}}{\frac{4}{7}} = -2$$

$$-\frac{21}{6} + \frac{3}{2} = -2 \rightarrow -\frac{21}{6} + \frac{9}{6} = -2$$

$$-\frac{12}{6} = -2 \rightarrow -2 = -2$$

4. **Example:** Solve $\rightarrow \frac{k+1}{3} - \frac{k}{5} = 3$.

$$\frac{k+1}{3} - \frac{k}{5} = 3$$

$$15\left(\frac{k+1}{3}\right) - 15\left(\frac{k}{5}\right) = 15(3)$$

$$5(k+1) - 3(k) = 45$$

$$5k + 5 - 3k = 45$$

$$2k + 5 = 45$$

$$2k = 40$$

$$k = 20$$

Multiply by each side by the LCD which is "15".

Check $\rightarrow \frac{20+1}{3} - \frac{20}{5} = 3$

$$\frac{21}{3} - \frac{20}{5} = 3$$

$$7 - 4 = 3$$

$$3 = 3$$

5. Example: Solve $\rightarrow \frac{6}{x} - \frac{9}{x-1} = \frac{1}{4}$

“x” cannot equal “0” or “1”.

$$\frac{6}{x} - \frac{9}{x-1} = \frac{1}{4}$$

$$4x(x-1)\frac{6}{x} - 4x(x-1)\frac{9}{x-1} = 4x(x-1)\frac{1}{4}$$

$$4(x-1)6 - 4x(9) = x^2 - x$$

$$24x - 24 - 36x = x^2 - x$$

$$0 = x^2 + 11x + 24$$

$$0 = (x+3)(x+8)$$

$$x = -3 \text{ or } -8$$

Multiply each side of the equation by the LCD which is $4x(x-1)$

Check $\rightarrow \frac{6}{-3} - \frac{9}{-3-1} = \frac{1}{4}$

$$-2 - \frac{9}{-4} = \frac{1}{4}$$

$$-2 + 2\frac{1}{4} = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{4}$$

Check $\rightarrow \frac{6}{-8} - \frac{9}{-8-1} = \frac{1}{4}$

$$-\frac{3}{4} + \frac{9}{9} = \frac{1}{4}$$

$$-\frac{3}{4} + 1 = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{4}$$

6. **Example:** solve $\rightarrow x - \frac{2}{x-3} = \frac{x-1}{3-x}$

“x” cannot equal 3

$$x - \frac{2}{x-3} = \frac{x-1}{3-x}$$

$$x - \frac{2}{x-3} = \frac{x-1}{-(x-3)}$$

$$x - \frac{2}{x-3} = -\frac{x-1}{(x-3)}$$

$$(x-3)x - (x-3)\frac{2}{x-3} = -\frac{x-1}{(x-3)}(x-3)$$

$$x(x-3) - 2 = -(x-1)$$

$$x^2 - 3x - 2 = -x + 1$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \text{ or } x = -1$$

Since “x” cannot equal 3, the only solution is $x = -1$

Multiply both sides of the equation by the LCD which is “x - 3”

Have students name the restrictions on the domain of an equation before solving it. Emphasize the importance of this when determining the solutions for an equation. In this example, the domain does not include 3. This limits the solutions to only -1.

Check $\rightarrow (-1) - \frac{2}{-1-3} = \frac{-1-1}{3--1}$

$$-1 + \frac{1}{2} = -\frac{1}{2}$$

$$-\frac{1}{2} = -\frac{1}{2}$$

7. **Example:** solve $\rightarrow \frac{2m}{m-1} + \frac{m-5}{m^2-1} = 1$

m cannot equal 1 or -1.

$$\frac{2m}{m-1} + \frac{m-5}{(m+1)(m-1)} = 1$$

$$(m-1)(m+1)\frac{2m}{m-1} + (m-1)(m+1)\frac{m-5}{(m+1)(m-1)} = (m+1)(m-1)(1)$$

$$2m(m+1) + (m-5) = m^2 - 1$$

$$2m^2 + 2m + m - 5 = m^2 - 1$$

$$m^2 + 3m - 4 = 0$$

$$(m+4)(m-1) = 0$$

$$m = -4 \text{ or } 1$$

Since "1" cannot be a solution then "m" must equal "-4"

Check: $\rightarrow \frac{2m}{m-1} + \frac{m-5}{(m+1)(m-1)} = 1 \rightarrow \frac{2(-4)}{-4-1} + \frac{-4-5}{(-4+1)(-4-1)} = 1$

$$\frac{-8}{-5} + \frac{-9}{(-3)(-5)} = 1 \rightarrow \frac{24}{15} - \frac{9}{15} = 1 \rightarrow \frac{15}{15} = 1$$

Name: _____

Date: _____

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SOLVING RATIONAL EQUATIONS WORKSHEET

Solve each equation and check (state excluded values).

1. $\frac{2a-3}{6} = \frac{2a}{3} + \frac{1}{2}$

6. $\frac{4x}{3x-2} + \frac{2x}{3x+2} = 2$

2. $\frac{2b-3}{7} - \frac{b}{2} = \frac{b+3}{14}$

7. $\frac{5}{5-p} - \frac{p^2}{5-p} = -2$

3. $\frac{3}{5x} + \frac{7}{2x} = 1$

8. $\frac{2a-3}{a-3} - 2 = \frac{12}{a+3}$

4. $\frac{5k}{k+2} + \frac{2}{k} = 5$

9. $\frac{2b-5}{b-2} - 2 = \frac{3}{b+2}$

5. $\frac{m}{m+1} + \frac{5}{m-1} = 1$

10. $\frac{4}{k^2-8k+12} = \frac{k}{k-2} + \frac{1}{k-6}$



SOLVING RATIONAL EQUATIONS WORKSHEET KEY

Solve each equation:

$$1. \quad \frac{2a-3}{6} = \frac{2a}{3} + \frac{1}{2}$$

$$6\left(\frac{2a-3}{6}\right) = 6\left(\frac{2a}{3}\right) + 6\left(\frac{1}{2}\right)$$

$$2a - 3 = 2(2a) + 3(1)$$

$$2a - 3 = 4a + 3$$

$$-3 = 2a + 3$$

$$-6 = 2a$$

$$-3 = a$$

Check:

$$\frac{2(-3)-3}{6} = \frac{2(-3)}{3} + \frac{1}{2}$$

$$\frac{-3}{2} = -2 + \frac{1}{2}$$

$$-1\frac{1}{2} = -1\frac{1}{2}$$

$$2. \quad \frac{2b-3}{7} - \frac{b}{2} = \frac{b+3}{14}$$

$$14\left(\frac{2b-3}{7}\right) - 14\left(\frac{b}{2}\right) = 14\left(\frac{b+3}{14}\right)$$

$$2(2b-3) - 7(b) = 1(b+3)$$

$$4b - 6 - 7b = b + 3$$

$$-9 = 4b$$

$$-\frac{9}{4} = b$$

Check:

$$2\left(\frac{-9}{4}\right) - 3 - \frac{-9}{2} = \frac{-9}{4} + 3$$

$$\frac{-9}{2} - 3 - \frac{-9}{2} = \frac{-9}{4} + 3$$

$$\frac{-15}{2} - \frac{-9}{2} = \frac{3}{4}$$

$$\frac{-15}{2} + \frac{9}{2} = \frac{3}{2}$$

$$\frac{3}{2} = \frac{3}{2}$$

"x" cannot equal "0"

$$3. \quad \frac{3}{5x} + \frac{7}{2x} = 1$$

$$10x\left(\frac{3}{5x}\right) + 10x\left(\frac{7}{2x}\right) = 10x(1)$$

$$2(3) + 5(7) = 10x$$

$$6 + 35 = 10x$$

$$41 = 10x$$

$$\frac{41}{10} = x$$

Check → $\frac{3}{5\left(\frac{41}{10}\right)} + \frac{7}{2\left(\frac{41}{10}\right)} = 1$

$$\frac{3}{205} + \frac{7}{82} = 1$$

$$\frac{30}{205} + \frac{70}{82} = 1$$

1 = 1

“k” cannot equal “-2”
or “0”

$$4. \frac{5k}{k+2} + \frac{2}{k} = 5$$

$$k(k+2) \frac{5k}{1(k+2)} + k(k+2) \frac{2}{k} = k(k+2)5$$

$$5k^2 + 2k + 4 = 5k^2 + 10k$$

$$2k + 4 = 10k$$

$$4 = 8k$$

$$\frac{1}{2} = k$$

$$\frac{5\left(\frac{1}{2}\right)}{\frac{1}{2} + 2} + \frac{2}{\frac{1}{2}} = 5$$
$$\frac{5}{\frac{5}{2}} + 4 = 5$$

$$5 = 5$$

$$5. \frac{m}{m+1} + \frac{5}{m-1} = 1$$

“m” cannot equal “-1” or “1”

$$(m+1)(m-1) \frac{m}{1(m+1)} + (m+1)(m-1) \frac{5}{1(m-1)} = (m+1)(m-1)1$$

$$(m-1)m + (m+1)(5) = (m+1)(m-1)$$

$$m^2 - m + 5m + 5 = m^2 - 1$$

$$4m + 5 = -1$$

$$4m = -6$$

$$m = -\frac{3}{2}$$

Check → $\frac{-\frac{3}{2}}{\frac{3}{2} + 1} + \frac{5}{-\frac{3}{2} - 1} = 1$

$$\frac{-\frac{3}{2}}{\frac{1}{2}} - \frac{5}{\frac{5}{2}} = 1$$

$$6 - 5 = 1$$

$$1 = 1$$

$$6. \frac{4x}{3x-2} + \frac{2x}{3x+2} = 2$$

“x” cannot equal $\frac{2}{3}$ or $-\frac{2}{3}$

$$(3x-2)(3x+2)\frac{4x}{1(3x-2)} + (3x-2)(3x+2)\frac{2x}{1(3x+2)} = (3x-2)(3x+2)2$$

$$(3x+2)4x + (3x-2)2x = 18x^2 - 8$$

$$12x^2 + 8x + 6x^2 - 4x = 18x^2 - 8$$

$$4x = -8$$

$$x = -2$$

Check → $\frac{4(-2)}{3(-2)-2} + \frac{2(-2)}{3(-2)+2} = 2$

$$\frac{-8}{-8} + \frac{-4}{-4} = 2$$

$$1 + 1 = 2$$

$$2 = 2$$

“p” is not equal to “5”

$$7. \frac{5}{5-p} - \frac{p^2}{5-p} = -2$$

Check → $\frac{5}{5-5} - \frac{(-5)^2}{5-5} = -2$

$$\frac{1}{2} - \frac{5}{2} = -2 \rightarrow -2 = -2$$

$$\frac{5-p^2}{5-p} = -2$$

$$(5-p)\frac{5-p^2}{1(5-p)} = (5-p)(-2)$$

$$5 - p^2 = -10 + 2p$$

$$0 = p^2 + 2p - 15$$

$$0 = (p+5)(p-3)$$

$$p = -5 \text{ or } 3$$

Check → $\frac{5-3^2}{5-3} = -2$

$$\frac{-4}{2} = -2$$

$$-2 = -2$$

$$8. \frac{2a-3}{a-3} - 2 = \frac{12}{a+3}$$

“a” cannot equal “3” or “-3”

$$(a-3)(a+3) \frac{2a-3}{1(a-3)} - (a-3)(a+3)2 = (a-3)(a+3) \frac{12}{1(a+3)}$$

$$(a+3)(2a-3) - 2a^2 + 18 = (a-3)(12)$$

$$2a^2 + 3a - 9 - 2a^2 + 18 = 12a - 36$$

$$3a + 9 = 12a - 36$$

$$45 = 9a$$

$$5 = a$$

Check → $\frac{2(5)-3}{5-3} - 2 = \frac{12}{5+3}$

$$\frac{7}{2} - 2 = \frac{12}{8}$$

$$\frac{3}{2} = \frac{3}{2}$$

$$9. \frac{2b-5}{b-2} - 2 = \frac{3}{b+2}$$

“b” cannot equal “2 or “-2”

$$(b-2)(b+2) \frac{2b-5}{1(b-2)} - (b-2)(b+2)2 = (b-2)(b+2) \frac{3}{1(b+2)}$$

$$(b+2)(2b-5) - 2b^2 + 8 = (b-2)3$$

$$2b^2 - b - 10 - 2b^2 + 8 = 3b - 6$$

$$-b - 2 = 3b - 6$$

$$4 = 4b$$

$$1 = b$$

Check → $\frac{2(1)-5}{1-2} - 2 = \frac{3}{1+2}$

$$\frac{-3}{-1} - 2 = \frac{3}{3}$$

$$1 = 1$$

$$10. \frac{4}{k^2 - 8k + 12} = \frac{k}{k - 2} + \frac{1}{k - 6}$$

“k” cannot equal “2” or “6”

$$(k - 2)(k - 6) \frac{4}{(k - 2)(k - 6)} = (k - 2)(k - 6) \frac{k}{1(k - 2)} + (k - 2)(k - 6) \frac{1}{1(k - 6)}$$

$$4 = (k - 6)k + (k - 2)1$$

$$4 = k^2 - 6k + k - 2$$

$$0 = k^2 - 5k - 6$$

$$0 = (k - 6)(k + 1)$$

$$k = 6 \text{ or } -1$$

Since “k” cannot equal “6” the solution is “-1”

Check → $\frac{4}{(-1)^2 - 8(-1) + 12} = \frac{-1}{-1 - 2} + \frac{1}{-1 - 6}$

$$\frac{4}{21} = \frac{1}{3} - \frac{1}{7}$$

$$\frac{4}{21} = \frac{4}{21}$$

Student Name: _____

Date: _____

SOLVING RATIONAL EQUATIONS CHECKLIST

1. On question 1, did the student solve the equation correctly and check solutions?
 - a. Yes (20 points)
 - b. Solved equation correctly but did not check solutions (15 points)
 - c. Equation was solved incorrectly but had only minor mathematical errors. Student did check solutions (10 points)
 - d. Equation was solved incorrectly and student did not check solutions (5 points)

2. On question 2, did the student solve the equation correctly and check solutions?
 - a. Yes (20 points)
 - b. Solved equation correctly but did not check solutions (15 points)
 - c. Equation was solved incorrectly but had only minor mathematical errors. Student did check solutions (10 points)
 - d. Equation was solved incorrectly and student did not check solutions (5 points)

3. On question 3, did the student solve the equation correctly and check solutions?
 - a. Yes (20 points)
 - b. Solved equation correctly but did not check solutions (15 points)
 - c. Equation was solved incorrectly but had only minor mathematical errors. Student did check solutions (10 points)
 - d. Equation was solved incorrectly and student did not check solutions (5 points)

4. On question 4, did the student solve the equation correctly and check solutions?
 - a. Yes (20 points)
 - b. Solved equation correctly but did not check solutions (15 points)
 - c. Equation was solved incorrectly but had only minor mathematical errors. Student did check solutions (10 points)
 - d. Equation was solved incorrectly and student did not check solutions (5 points)

5. On question 5, did the student solve the equation correctly and check solutions?
 - a. Yes (20 points)
 - b. Solved equation correctly but did not check solutions (15 points)
 - c. Equation was solved incorrectly but had only minor mathematical errors. Student did check solutions (10 points)
 - d. Equation was solved incorrectly and student did not check solutions (5 points)

6. On question 6, did the student solve the equation correctly and check solutions?
- Yes (20 points)
 - Solved equation correctly but did not check solutions (15 points)
 - Equation was solved incorrectly but had only minor mathematical errors. Student did check solutions (10 points)
 - Equation was solved incorrectly and student did not check solutions (5 points)
7. On question 7, did the student solve the equation correctly and check solutions?
- Yes (20 points)
 - Solved equation correctly but did not check solutions (15 points)
 - Equation was solved incorrectly but had only minor mathematical errors. Student did check solutions (10 points)
 - Equation was solved incorrectly and student did not check solutions (5 points)
8. On question 8, did the student solve the equation correctly and check solutions?
- Yes (20 points)
 - Solved equation correctly but did not check solutions (15 points)
 - Equation was solved incorrectly but had only minor mathematical errors. Student did check solutions (10 points)
 - Equation was solved incorrectly and student did not check solutions (5 points)
9. On question 9, did the student solve the equation correctly and check solutions?
- Yes (20 points)
 - Solved equation correctly but did not check solutions (15 points)
 - Equation was solved incorrectly but had only minor mathematical errors. Student did check solutions (10 points)
 - Equation was solved incorrectly and student did not check solutions (5 points)
10. On question 10, did the student solve the equation correctly and check solutions?
- Yes (20 points)
 - Solved equation correctly but did not check solutions (15 points)
 - Equation was solved incorrectly but had only minor mathematical errors. Student did check solutions (10 points)
 - Equation was solved incorrectly and student did not check solutions (5 points)

Total Number of Points _____

- A 180 points and above
- B 160 points and above
- C 140 points and above
- D 120 points and above
- F 119 points and below

