1. Solving quadratics using the table method

Factorize the following expression by using table method

A = $x^2 + 7x + 12$

.	**x**	**4**
x	x^2	4x
3	3x	12

Dot means multiplication, simply you multiply x by x, 4 by x, …, etc . You add them up together and you will obtain the original expression. Therefore factorization is

$x^2 + 7x + 12 = (x + 4)(x + 3)$

Similarly: factorize the following expression by using table method

$x^2 - x - 12 = (x - 4) (x + 3)$

.	**x**	**- 4**
x	x^2	- 4x
3	3x	**-12**

2. Solving quadratics by completing square

<u>Exercise 1.</u>

Does the curve y = $x^2 + 6x - 7$ have maximum or minimum? Find the roots and sketch the graph.
(Roots are the intersections of the curve with the x-axis)

For calculating the **roots** of quadratic equation we can use the table method or method of completing square:

TABLE METHOD

.	**x**	**7**
x	x^2	7x
-1	-1x	**-7**

COMPLETING SQUARE

$x^2 + 6x - 7 = (x+3)^2 - 9 - 7 = 0$

$(x+3)2=16$

$x+3 = \pm\sqrt{16}$

$\mathbf{x_1 = \sqrt{16} + 3 = 7}$

$\mathbf{x_2 = -\sqrt{16} + 3 = -4 + 3 = -1}$

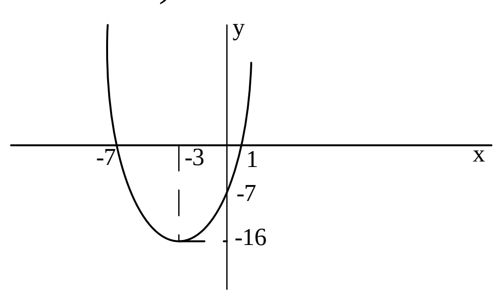
Now, we can write the quadratic equation as the multiplication of two factors **(x+7)(x-1) = 0**, where the roots are numbers: –7 and 1

Because we know that a > 0 (*positive*) the curve is convex, so it has the minimum point with the coordinates [x, y]. To find this coordinates, we are using the method of completing the square again.

$y = x^2 + 6x - 7 = (x + 3)^2 - 9 - 7 = (x + 3)^2 - 16$

(After expanding the brackets you will obtain $x^2 + 6x + 9 - 16 = x^2 + 6x - 7$)

The value is –16 for the x = -3 (MIN [-3, -16])



Exercise 2.

Does the curve y = - x² - 4x, have maximum or minimum? Find the roots and sketch the graph.

For calculating the roots we can use one of the following three methods:

1. FACTORISING

$-x^2 - 4x = 0$

$-x (x+4) = 0$

When the multiplication of two factors is zero? If one of the factor equals zero.

$-x = 0 \Rightarrow \mathbf{x_1 = 0}$

$(x+4) = 0 \Rightarrow \mathbf{x_2 = -4}$

2. COMPLETING SQUARE

$-(x^2 + 4x) = -(x + 2)^2 + 4 = 0$

$-(x + 2)^2 = -4$

$(x + 2)^2 = 4$

$x + 2 = \pm\sqrt{4}$

$\mathbf{x_1 = \sqrt{4} - 2 = 0}$

$\mathbf{x_2 = -\sqrt{4} - 2 = -4}$

3. Solving quadratics using formulas

3. USING FORMULAS

$\mathbf{x_1} = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ $\mathbf{x_2} = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$, **where** $b^2 - 4ac$ **is called Discriminate**

In the above exercise

$\mathbf{x_1} = \dfrac{4+\sqrt{16-4(-1).0}}{2(-1)} = \dfrac{4+4}{(-2)} = -4$ $\mathbf{x_2} = \dfrac{-b-\sqrt{b^2-4ac}}{2a} = \dfrac{4-4}{(-2)} = 0$

Using all these four methods (factorizing, completing square, table method or using formulas) you MUST reach the same roots. It is just up to you, which method you choose.

Because we know that a < 0 the curve is concave, so it has the maximum point with the coordinates [x, y] To find this coordinates, we are using the method of completing the square again.

$-y = x^2 + 4x = (x + 2)^2 - 4$

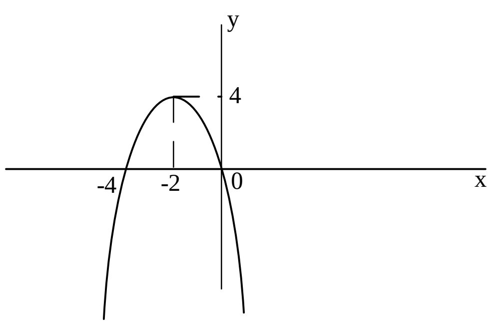
(After expanding brackets you will get $-y = x^2 - 4x + 4 - 4$*)*

We can see that if *x* = -2, the value *y* for this *x* is 4 (Because when we substitute –2 for the *x*, we will get:

$-y = (-2 + 2)^2 - 4 = 0^2 - 4 = -4 \Rightarrow -y = -4 \Rightarrow y = 4$

The maximum point has coordinates [x, y]. The value is 4 for the x = -2

MAX [-2, 4]



VIETE'S FORMULAS

Assuming that the quadratic equation has two distinct roots α, β. Find the relationship between the coefficients a, b, c and roots α, β.

$ax^2 + bx + c = 0 \quad a, b, c \neq 0$

Divide the equation all the way through by a.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

If α and β are two distinct roots of that equation, we may write them as two factors in the following form.

$$(x-a)(x-b) = 0$$
$$x^2 - ax - bx + ab = 0$$
$$x^2 - x(a+b) + ab = 0$$

1. $-x(a+b) = \left(\frac{b}{a}\right)x$ and after simplifying we obtain

2. $ab = \frac{c}{a}$

This is called Viete's formula

$$a + b = -\left(\frac{b}{a}\right)$$
$$ab = \frac{c}{a}$$

EXERCISE 1.

Write down the equation having: $a + b = 2$

$ab = -5$

We already know **$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$** and by using Viete`s formula we can get:

$x^2 - 2x - 5 = 0$ This is the equation we were looking for.

We can find the roots:

$$a = \frac{2+\sqrt{4-4.1.(-5)}}{2.1} = \frac{2+\sqrt{24}}{2} = \frac{2+\sqrt{6.4}}{2} = \frac{2+2\sqrt{6}}{2} = 1+\sqrt{6}$$

$$b = \frac{2-\sqrt{4-4.1.(-5)}}{2.1} = \frac{2-\sqrt{24}}{2} = \frac{2-\sqrt{6.4}}{2} = \frac{2-2\sqrt{6}}{2} = 1-\sqrt{6}$$

$$ab = (1+\sqrt{6})(1-\sqrt{6}) = 1-\sqrt{6}+\sqrt{6}-6 = -5$$
$$a + b = (1+\sqrt{6}) + (1-\sqrt{6}) = 2$$

EXERCISE 2

If α and β are the roots of the quadratic equation $x^2 - 4x + 2 = 0$ find the quadratic equation that has roots of $\mathbf{x_1 = 1/\alpha^2,\ x_2 = 1/\beta^2}$

$$a = \frac{4+\sqrt{16-4.1.2}}{2.1} = \frac{4+\sqrt{8}}{2} = \frac{4+\sqrt{2.4}}{2} = \frac{4+2\sqrt{2}}{2} = 2+\sqrt{2}$$

$$b = \frac{4-\sqrt{16-4.1.2}}{2.1} = \frac{4-\sqrt{8}}{2} = \frac{4-\sqrt{2.4}}{2} = \frac{4-2\sqrt{2}}{2} = 2-\sqrt{2}$$

$$\mathbf{x_1} = \frac{1}{a^2} = \frac{1}{\left(2+\sqrt{2}\right)^2} = \frac{1}{4+4\sqrt{2}+2} = \frac{1}{6+4\sqrt{2}}$$

$$\mathbf{x_2} = \frac{1}{b^2} = \frac{1}{\left(2-\sqrt{2}\right)^2} = \frac{1}{4-4\sqrt{2}+2} = \frac{1}{6-4\sqrt{2}}$$

From the Viete`s formulas we know:

$$-\left(\frac{b}{a}\right) = x_1 + x_2 = \frac{1}{6+4\sqrt{2}} + \frac{1}{6-4\sqrt{2}} = \frac{6-4\sqrt{2}+6+4\sqrt{2}}{\left(6+4\sqrt{2}\right).\left(6-4\sqrt{2}\right)} = \frac{12}{36-16.2} = \frac{12}{4} = 3$$

$$\frac{c}{a} = x_1.x_2 = \frac{1}{6+4\sqrt{2}}.\frac{1}{6-4\sqrt{2}} == \frac{1}{36-16.2} = \frac{1}{4}$$

We already know $\mathbf{x^2 + \frac{b}{a}x + \frac{c}{a} = 0}$ and by using Viete`s formula we can get:

$\mathbf{x^2 - 3x - \frac{1}{4} = 0}$ or $\mathbf{4x^2 - 12x - 1 = 0}$ This is the equation we were looking for.

EXERCISE 3

The expression: $p + qx - x^2$ has a maximum value + 5 when x = 2. Find the numerical values of p and q.

We will start by using the method of completing square:

$$y = -x^2 + qx + p$$

$$-y = x^2 - qx - p$$

$$-y = \left(x - \frac{q}{2}\right)^2 - \frac{q^2}{4} - p$$

$$-y = \left(x - \frac{q}{2}\right)^2 - \frac{q^2 + 4p}{4}$$

Now we know the followings:

1) $\frac{q}{2} = 2 \Rightarrow q = 4$ and 2) $\frac{q^2 + 4p}{4} = 5$

$$\frac{4^2 + 4p}{4} = 5$$

$$4p = 20 - 16 \Rightarrow p = 1$$

Conclusion: When q = 4 and p = 1 the expression is: $-x^2 + 4x + 1$.