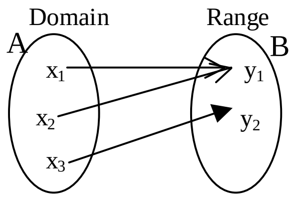
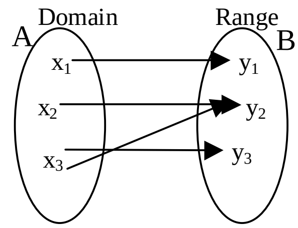
FUNCTION AND PROPERTIES OF FUNCTION – SUMMARY

WHAT IS FUNCTION?
Function is the relationship of two sets (A, B), such that for each element in A there exists only one image in the set B. ($A, B \in R$)



FUNCTION



NOT A FUNCTION – there are 2 images in set B for one point in the set A (ONE TO MANY)

DEFINITION OF THE FUNCTION
Every set of ordered pairs $[x, y] \in R \times R$ for which is valid: for each element $x \in R$ there exists **just one** $y \in R$, so that $[x, y] \in f(x)$.

DOMAIN OF A FUNCTION – we denote set A as *D(f)* and we call it the **DOMAIN** of the function.
Notation: $D(f) = \{x \in R;\ \exists y \in R;\ y \in f(x)\}$

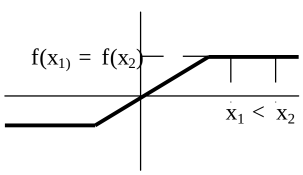
RANGE OF A FUNCITON- we denote set B as *R(f)* and call it the **RANGE** of a function *f(x)*. For each element in set B there exists at least one element in set A.
(In Slovakia we denote R(f) as H(f) – obor hodnôt)
Notation: $R(f) = \{y \in R;\ \exists x \in R;\ y \in f(x)\}$

PROPERTIES OF THE FUNCITON

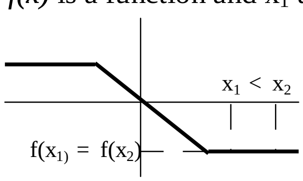
INCREASING FUNCTION (rastúca)
f(x) is a function and x_1 and $x_2 \in D(f)$; *f(x)* is **increasing** if $x_1 < x_2$ implies that $f(x_1) < f(x_2)$

DECREASING FUNCTION (klesajúca)
f(x) is a function and x_1 and $x_2 \in D(f)$; *f(x)* is **decreasing** if $x_1 < x_2$ implies that $f(x_1) > f(x_2)$

NON-DECREASING FUNCTION (neklesajúca)
f(x) is a function and x_1 and $x_2 \in D(f)$; *f(x)* is **non-decreasing** if $x_1 < x_2$ implies that $f(x_1) \leq f(x_2)$



NON-INCREASING FUNCTION (nerastúca)
f(x) is a function and x_1 and $x_2 \in D(f)$; *f(x)* is **non-increasing** if $x_1 < x_2$ implies that $f(x_1) \geq f(x_2)$



MANY-TO-ONE FUNCTION= non-decreasing or non-increasing function or each function for which is valid that there exist at least two x with one f(x).

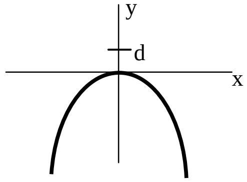
ONE-TO-ONE FUNCTION (prostá)

f(x) is a **one-to-one** function if for all x_1 and $x_2 \in$ *D(f)*;such that $x_1 \neq x_2$ *f(x)* implies that that $f(x_1) \neq f(x_2)$. (Exponential function is an example of one-to-one function)

(LINE TEST – function is one-to-one if and only if each horizontal line intersects the graph of the function just in one point)

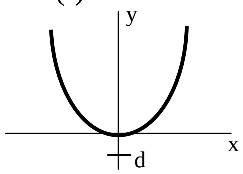
BOUNDED ABOVE FUNCTION (ohraničená z hora)

f(x) is a function and A∈ *D(f)*; *f(x)* is **bounded above** if $\exists d \in R$ such that $f(x) \leq d$ for all $x \in D(f)$



BOUNDED BELOW FUNCTION (ohraničená z dola)

f(x) is a function and A∈ *D(f)*; *f(x)* is **bounded below** if $\exists d \in R$ such that $f(x) \geq d$ for all $x \in D(f)$

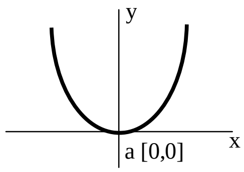


BOUNDED

Function is bounded if it is bounded below and bounded above as well.

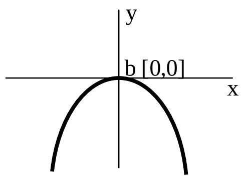
MINIMUM OF THE FUNCITON

Let $A \subseteq D(f)$ and $a \in A$; the function has **minimum** at point $a \in A$ if for all $x \in A$ $f(x) \geq f(a)$ (we consider f(x)>0)



MAXIMUM OF THE FUNCITON

Let $A \subseteq D(f)$ and $b \in A$; the function has **maximum** at point $b \in A$ if for all $x \in A$ $f(x) \leq f(b)$ (we consider f(x)<0)



EVEN FUNCTION (párna)

Function f is **even** just then if it is valid that for each $x \in D(f)$ exists also $-x \in D(f)$ such that $f(-x) = f(x)$.

Function is symmetric around the y-axis. (Example of even function is $y = x^2$)

ODD FUNCTION (nepárna)

Function f is **odd** just then if it is valid for each $x \in D(f)$ exists also $-x \in D(f)$ such that $f(-x) = -f(x)$.

Function is symmetric around the origin. (Example of odd function is $y = x$)

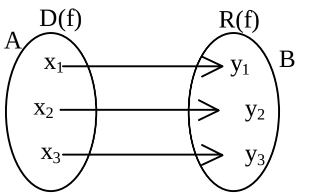
INVERSE FUNCTION

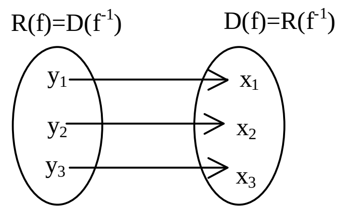
Let $f(x)$ to be an **one-to-one** function with D(f) and R(f); then we can find the **inverse function** to this function if for each $y \in R(f)$ we can assign $x \in D(f)$ for which $f(x) = y$

Notation of inverse function $f^{-1}(x)$, $g(x)$

Function and inverse function are symmetric around the line $y = x$

FUNCTION $f(x)$ **INVERSE FUNCTION $f^{-1}(x)$**





If function f(x) is one-to-one function it has inverse function, which is one-to-one as well.
If function f(x) is not one-to-one function is doesn't have inverse function.

<u>EXAMPLE</u>

Imagine a free fall and you want to know the distance in time 0; 1; 1,5…4 seconds. The formula for free fall is $s = \frac{1}{2} g.t^2$ (g = 10 ms^{-2}). We can see it's quadratic function, so the graph is parable.

t [s]	0	1	1,5	2	2,5	3	3,5	4
s [m]	0	5	11,25	20	31,25	45	61,25	80

Time t is the variable that implies D(h) = $\langle 0,4 \rangle$

Distance s creates the Range of this function R(h) = $\langle 0,80 \rangle$

h: $s = \frac{1}{2} g.t^2$

The inverse function to function h is:

s [m]	0	1,25	5	11,25	20	31,25	45	61,25	80
t [s]	0	0,5	1	1,5	2	2,5	3	3,5	4

h^{-1}: $t = \frac{1}{2} g.s^2 \Rightarrow 2t = g.s^2 \Rightarrow \frac{2t}{g} = s^2$

$s = \pm\sqrt{\frac{2.t}{g}}$ (we can't have negative distance, so we consider only positive)

D(h^{-1}) = $\langle 0,80 \rangle$

R(f) = $\langle 0,4 \rangle$