INTEGRATION

Primitive function – We are given a function *f* so that (a, b) $\subset$ D(f). A primitive function on the interval (a, b) is a function F, for which it is true that F' (x) = f(x)

E.g.: $(x^2)' = 2x$ then function F: $y = x^2$ is a primitive function to the function f(x) = 2x on the interval $(-\infty, \infty)$

For any two primitive functions G, F to the function f on (a, b) it is valid that G(x) = F(x) + c, where c $\in$ R.

Notation: We will notate the primitive functions F to the function f on (a, b) as

$\int f(x)\,dx = F(x)$

We read it: integral of a function f,

f(x) = integrand = integrated function

dx notates, what it a variable in the function; it is called a differential

Integration means to look for a primitive function; it is an opposite operation to differentiation.

To integrate certain basic expressions we normally use the following formulae for integration:

y = 0	$\int 0\,dx = c$	$c \in R$
y = 1	$\int 1\,dx = x + c$	$x \in R$
$y = x^n$	$\int x^n\,dx = \frac{x}{n+1}$	$x \in R$
$y = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$	$x \neq 0$
y = sin x	$\int \sin x\,dx = -\cos x + c$	
y = cos x	$\int \cos x\,dx = \sin x + c$	
$y = e^x$	$\int e^x\,dx = e^x + c$	

INTEGRATION RULES:

$\int k \cdot f(x)\,dx = k \cdot \int f(x)\,dx$

$\int [\,f(x) \pm g(x)\,]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

FUNDAMENTAL METHODS OF INTEGRATION

SUBSTITUTION

Substitution, as one of the methods of integration is used, when there is a complex function as an integrand and there occurs also derivation of its inner component. Then we substitute for this inner part, and differentiate. After this step, we return back under the integral and integrate, e.g.

$\int \underline{2x} \sin x^2 \, \underline{dx} =$ $\quad t = x^2$ $\quad = \int \sin t \, dt = -\cos t + c = -\cos x^2 + c$

$\quad dt = 2x \, dx$

PER PARTES = INTEGRATION BY PARTS

$(u \cdot v)' = u' \cdot v + u \cdot v'$

$u \cdot v' = (u \cdot v)' - u' \cdot v$

$\int u \cdot v' \, dx = \int (u \cdot v)' \, dx - \int u' \cdot v \, dx$

$\int u \cdot v' \, dx = (u \cdot v) - \int u' \cdot v \, dx$

E.g. $\int x \cdot e^x \, dx =$ $\quad u = x \quad u' = 1$ $\quad = x \cdot e^x - \int e^x \, dx = \underline{x \cdot e^x - e^x + c}$

$\quad v' = e^x \quad v = e^x$

THE AREA UNDER A CURVE and DEFINITE INTEGRAL

Suppose A is the area bounded by the curve y = f(x), the x – axis and the lines x = a and x = b. We say that A is the area under the curve from x = a to x = b and notate:

$A = \int_a^b y \, dx$, where y = f(x).

This integral is known as a ***definite*** integral because the limits of integration, i.e. x= a and x = b are known.

We calculate this integral by applying the *Newton - Leibniz rule* as follows:

if $\int f(x)dx = F(x) + c$, then

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) + c - (F(a) + c) = F(b) - F(a)$$

We see that the constants of integration cancel out so that in the case of a definite integral there is no need to give an arbitrary constant in the result.

CALCULATION OF THE AREA UNDER A CURVE

When we calculate the area under a curve, the important first step is to make a sketch of the curve. We must then remember that an area lying 'above' the x-axis will have a positive value, whereas areas lying 'below' the x-axis will be negative.

AREA DEFINED BY TWO CURVES

An area may be defined by two curves and in this case it is essential to make a sketch to determine the points of intersection of the two curves.

Suppose the curves y = f(x) and y = g(x) intersect at the points where x = a and x = b.

The area between the curve y = f(x) and the x-axis from x = a to x = b is given by

$$\int_a^b f(x)dx$$

The area between the curve y = g(x) and the x-axis from x = a to x = b is given by

$$\int_a^b g(x)dx$$

The area between the two curves is then

$$\int_a^b f(x)dx - \int_a^b g(x)dx = \int_a^b [f(x) - g(x)]\, dx$$

VOLUME OF ROTATION

The volume of a solid generated by rotating about the x – axis, that part of the curve y = f(x), between the lines x = a and x = b, is given by

$$V = \pi \int_a^b f^2(x)dx$$