INEQUALITIES

Solving Inequalities by Using Addition and Subtraction

You can use the addition and subtraction properties for inequalities when solving problems involving inequalities.

Addition and Subtraction Properties for Inequalities
For all numbers a, b, and c, **1.** if $a > b$, then $a + c > b + c$ and $a - c > b - c$; **2.** if $a < b$, then $a + c < b + c$ and $a - c < b - c$.

Example: Solve $3a + 5 > 4 + 2a$.

$$3a + 5 > 4 + 2a$$
$$3a - 2a + 5 > 4 + 2a - 2a$$
$$a + 5 > 4$$
$$a + 5 - 5 > 4 - 5$$
$$a > -1$$

The solution set is {all numbers greater than -1}.

To check your solution, choose two numbers, one greater than -1, and one less than -1. Substitute both numbers in the original inequality. Only those numbers greater than -1 should yield a true statement.

The solution set in the above example, written in set-builder notation is $\{a \mid a > -1\}$. This is read "The set of all numbers a such that a is greater than -1."

Solving Inequalities by Using Multiplication and Division

You can solve inequalities by using the same methods you have already used to solve equations. However, when solving inequalities, if you multiply or divide each side by the same negative number, you must *reverse* the direction of the inequality symbol. The following chart shows the multiplication and division properties for solving inequalities.

Multiplication Property for Inequalities	Division Property for Inequalities
For all numbers a, b, and c, **1.** if c is positive and $a < b$, then $ac < bc$; if c is positive and $a > b$, then $ac > bc$; **2.** if c is negative and $a < b$, then $ac > bc$; if c is negative and $a > b$, then $ac < bc$.	For all numbers a, b, and c, **1.** if c is positive and $a < b$, then $\frac{a}{c} < \frac{b}{c}$; if c is positive and $a > b$, then $\frac{a}{c} > \frac{b}{c}$; **2.** if c is negative and $a < b$, then $\frac{a}{c} > \frac{b}{c}$; if c is negative and $a > b$, then $\frac{a}{c} < \frac{b}{c}$.

Solving Multi-Step Inequalities

Solving an inequality may require more than one operation. Use the same procedure you used for solving equations to solve inequalities.

Procedure For Solving Inequalities
1. Use the distributive property to remove any grouping symbols. **2.** Simplify each side of the inequality. **3.** Undo any indicated additions and subtractions. **4.** Undo any indicated multiplications and divisions involving the variable.

Example: Solve $21 > -7(m + 2)$.

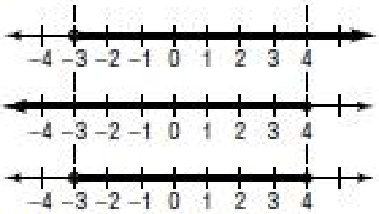
$$21 > -7(m + 2)$$
$$21 > -7m - 14$$ **Use the distributive property.**
$$21 + 14 > -7m - 14 + 14$$ **Subtraction is indicated; use addition.**
$$35 > -7m$$
$$\frac{35}{-7} < \frac{-7m}{-7}$$ **Multiplication is indicated; use division. Reverse the inequality symbol.**
$$-5 < m$$

The solution set is $\{m \mid -5 < m\}$, or $\{m \mid m > -5\}$.

Solving Compound Inequalities

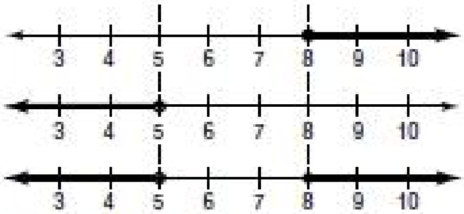
A **compound inequality** consists of two inequalities that are connected by the words *and* or *or*. A compound inequality containing *and* is true only if *both* inequalities are true. Its graph is the **intersection** of the graphs of the two inequalities. A compound inequality containing *or* is true if one or more of the inequalities is true. Its graph is the **union** of the graphs of the two inequalities.

Example 1: $x > -3$ and $x \le 4$



The solution set, shown in the bottom graph, is $\{x \mid -3 < x \le 4\}$.

Example 2: $t \ge 8$ or $t < 5$



The solution set is $\{t \mid t \ge 8 \text{ or } t < 5\}$.

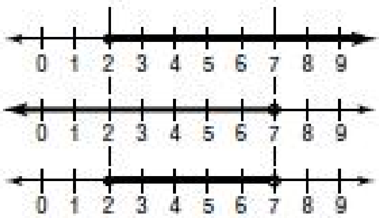
Sometimes it is better to first solve each inequality and then graph the solution. Study the examples below.

Example 3: $-3 \le p - 5 < 2$

$-3 \le p - 5$ and $p - 5 < 2$

$-3 + 5 \le p - 5 + 5$ $\quad$ $p - 5 + 5 < 2 + 5$

$2 \le p$ $\quad$ $p < 7$



The solution set is $\{p \mid 2 \le p < 7\}$.

Example 4: $2a + 1 < 11$ or $a > 3a + 2$

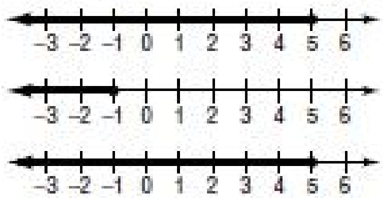
$2a + 1 < 11$ or $a > 3a + 2$

$2a + 1 - 1 < 11 - 1$ $\quad$ $a - 3a > 3a - 3a + 2$

$2a < 10$ $\quad$ $-2a > 2$

$\frac{2a}{2} < \frac{10}{2}$ $\quad$ $\frac{-2a}{-2} > \frac{2}{-2}$

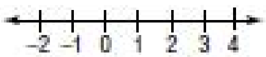
$a < 5$ $\quad$ $a < -1$



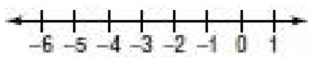
The solution set is $\{a \mid a \le 5\}$.

Graph the solution set of each compound inequality.

1. $b > -1$ and $b \le 3$



2. $y \le -4$ or $y > 0$

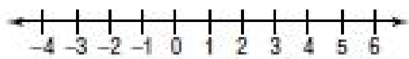


3. $2 \ge q \ge -5$

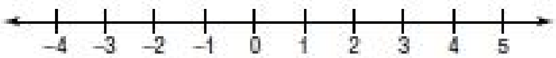
Solve each compound inequality. Then graph the solution set.

4. $2x + 4 \le 6$ or $x \ge 2x - 4$

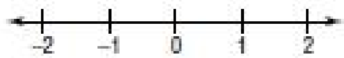
5. $d - 3 < 6d + 12 < 2d + 32$



6. $4(g - 3) + 2 < 6$ and $7g > 3(2g - 1)$



7. $3a + 2 \ge 5$ or $7 + 3a < 2(a + 3)$



A) **Solve each inequality. Then check your solution.**

1. $a + 4 < 14$
2. $9k - 12 > 80 + 8k$
3. $-19 + x < 2x - 33$
4. $6y > 14 - 2 + 7y$
5. $3n + 17 < 4n - 6$
6. $\frac{3}{2}q - \frac{25}{5} \geq \frac{2q}{4}$
7. $h + \frac{2}{3} \leq 2 - \frac{2}{3}$
8. $4p - 3.2 \geq 3p + 0.7$
9. $-2\frac{1}{2}z \leq 3\frac{1}{3} + 2\frac{1}{3} - 3\frac{1}{2}z$

B)

Define a variable, write an inequality, and solve each problem. Then check your solution.

13. Four times a number is no more than 108.
14. A number divided by 5 is at least -10.

C)

Define a variable, write an inequality, and solve each problem. Then check your solution.

19. A number decreased by 10 is greater than -5.
20. A number increased by 2 is at most 6.
21. A number increased by -1 is less than 10.
22. A number decreased by -4 is at least 9.

D) **Solve each inequality. Then check your solution.**

1. $11y + 13 \geq -1$
2. $-3v + 3 \leq -12$
3. $\frac{q}{7} + 1 > -5$
4. $-1 - \frac{m}{4} \leq 5$
5. $\frac{3x}{7} - 2 < -3$
6. $\frac{4x - 2}{5} \geq -4$
7. $9n - 24n + 42 > 0$
8. $4.6(x - 3.4) \geq 5.1x$
9. $7.3y - 3.02 > 4.9y$
10. $6y + 10 > 8 - (y + 14)$
11. $m + 17 \leq -(4m - 13)$
12. $-5x - (2x + 3) \geq 1$

10. $\frac{1}{b} + 4 \leq 10 + \frac{1}{b}, b \neq 0$
11. $6r > 10r - r - 3r$
12. $3.2x < 2x - (9 - 1.2x)$