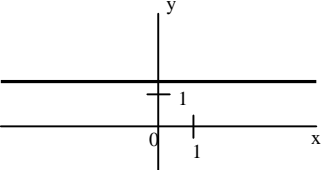
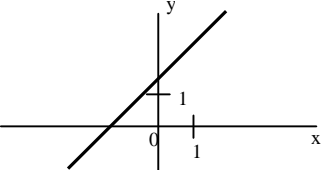
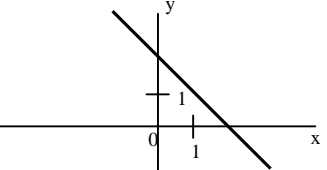


**1. LINEAR FUNCTION**  $y = ax + b$ ,  $a$  = gradient,  $b$ - interception with  $y$ -axis

$$(a = \frac{y_2 - y_1}{x_2 - x_1})$$

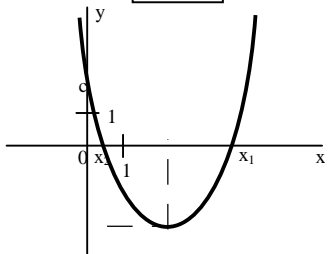
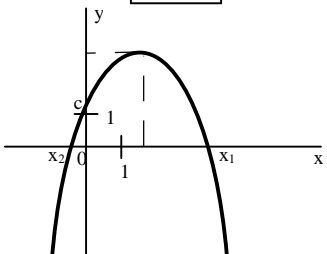
<b>a = 0</b>	<b>a &gt; 0</b>	<b>a &lt; 0</b>
		
<ul style="list-style-type: none"> <li>- Domain = <math>\mathbb{R}</math></li> <li>- Range = <math>\{b\}</math></li> <li>- It's many to one,</li> <li>- It's not one to one and therefore it's not increasing neither decreasing</li> <li>- Bounded</li> <li>- In each point <math>x \in \mathbb{R}</math> there is maximum and minimum</li> </ul>	<ul style="list-style-type: none"> <li>- Domain = <math>\mathbb{R}</math></li> <li>- Range = <math>\mathbb{R}</math></li> <li>- It's one to one</li> <li>- It's increasing</li> <li>- Not bounded above, neither bounded below</li> <li>- It has no maximum neither minimum</li> </ul>	<ul style="list-style-type: none"> <li>- Domain = <math>\mathbb{R}</math></li> <li>- Range = <math>\mathbb{R}</math></li> <li>- It's one to one</li> <li>- It's decreasing</li> <li>- Not bounded above, neither bounded below</li> <li>- It has no maximum neither minimum</li> </ul>

**2. QUADRATIC FUNCTION**  $y = ax^2 + bx + c$ , the graph of this function is **parabole**

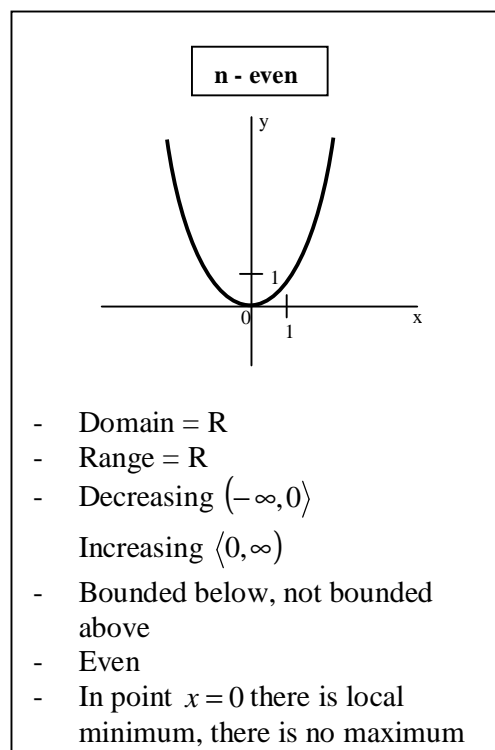
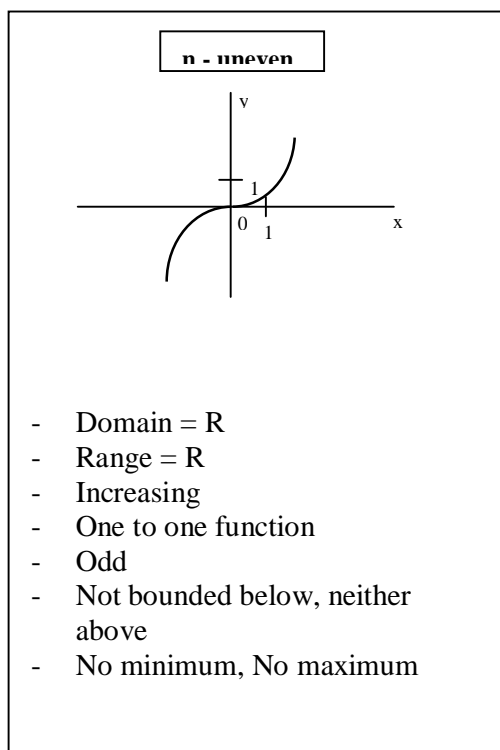
Useful formulas: Discriminant  $D = b^2 - 4ac$       Roots:  $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

MAX/MIN

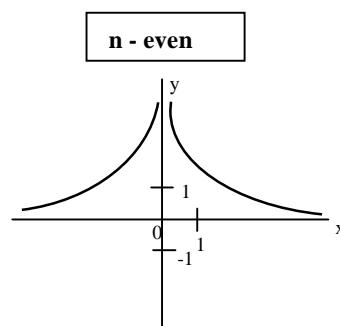
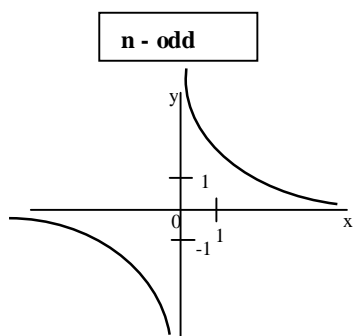
$$\left[ -\frac{b}{2a}, c - \frac{b^2}{4a} \right]$$

<b>a &gt; 0</b>	<b>a &lt; 0</b>
	
<ul style="list-style-type: none"> <li>- Domain = <math>\mathbb{R}</math> – many to one</li> <li>- Range = <math>\left\langle c - \frac{b^2}{4a}, \infty \right\rangle</math></li> <li>- Decreasing <math>\left( -\infty, -\frac{b}{2a} \right)</math></li> <li>- Increasing <math>\left( -\frac{b}{2a}, \infty \right)</math></li> <li>- Bounded below, not bounded above</li> <li>- In point <math>x = -\frac{b}{2a}</math> there is local minimum</li> </ul>	<ul style="list-style-type: none"> <li>- Domain = <math>\mathbb{R}</math> – many to one</li> <li>- Range = <math>\left( -\infty, c - \frac{b^2}{4a} \right)</math></li> <li>- Increasing <math>\left( -\infty, -\frac{b}{2a} \right)</math></li> <li>- Decreasing <math>\left( -\frac{b}{2a}, \infty \right)</math></li> <li>- Bounded above, not bounded below</li> <li>- In point <math>x = -\frac{b}{2a}</math> there is local maximum</li> </ul>

### 3. FUNCTION WITH NATURAL EXPONENT $y = x^n; n \in \mathbb{N}$



### 4. FUNCTION WITH NEGATIVE EXPONENT $y = x^{-n}; n \in \mathbb{N}$



Domain =  $\mathbb{R} - \{0\}$

- Range =  $\mathbb{R} - \{0\}$
- Decreasing on the intervals  $(-\infty, 0), (0, \infty)$
- Not bounded above neither below
- Odd

- Range =  $\mathbb{R}^+$
- Decreasing on the interval  $(-\infty, 0)$
- Increasing on the interval  $(0, \infty)$
- Not bounded above,
- Bounded below
- Even

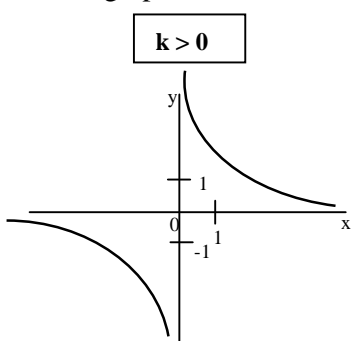
No maximum, no minimum

**5. RATIONAL FRACTIONAL FUNCTION**  $y = \frac{P(x)}{Q(x)} = \frac{ax+b}{cx+d} = \frac{k}{x}; (k \neq 0)$

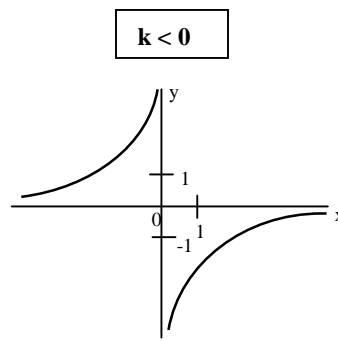
$P(x)$  and  $Q(x)$  are polynomials and  $Q(x) \neq 0$

$a, b, c, d \in R; c, d \neq 0$  and  $ad \neq bc$ ; (if  $ad = bc$  then it is constant function)

- The graph of this function is **hyperbola**



Decreasing on the intervals  
 $(-\infty, 0), (0, \infty)$

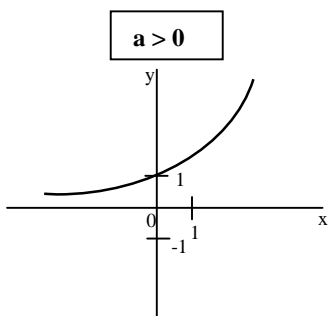


Increasing on the intervals  
 $(-\infty, 0), (0, \infty)$

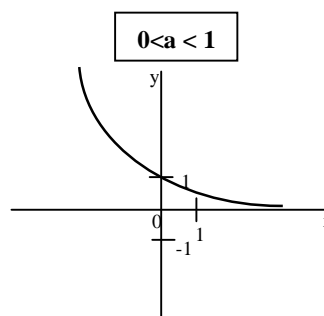
Domain =  $R - \{0\}$   
Range =  $R - \{0\}$

Not bounded above neither below  
No maximum, no minimum  
Odd

**6. EXPONENTIAL FUNCTION**  $y = a^x$  (conditions:  $a > 0; a \neq 1; x \in R$ )



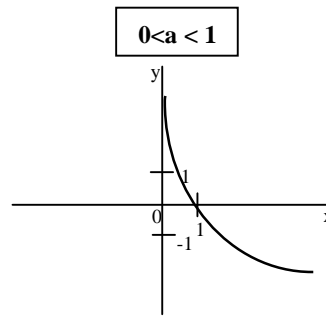
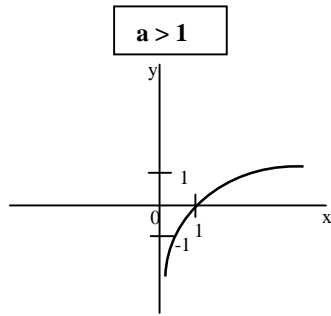
Increasing and one to one



Decreasing and one to one

Domain =  $R$   
Range =  $(0, \infty)$   
Bounded below, not bounded above  
No minimum, no maximum  
The image in point  $x = 0$  is 1

## 7. LOGARITHMIC FUNCTION $y = \log_a x$



Domain =  $(0, \infty)$

Range =  $\mathbb{R}$

Increasing and one to one

Decreasing and one to one

Not bounded below, not bounded above

No minimum, no maximum

The image in point  $x = 1$  is 0

An exponential function with the base e (Euler's number) is called **natural logarithmic function**  
 $y = \ln x$

## 8. MODULUS FUNCTION

We will use our knowledge about the linear function.

Example: Draw graph of the following function  $y = |2 - x| + |2x - 7| + 3|1 + x| - 15$ ;

where  $x \in \langle -3, 5 \rangle$

1. find the critical points

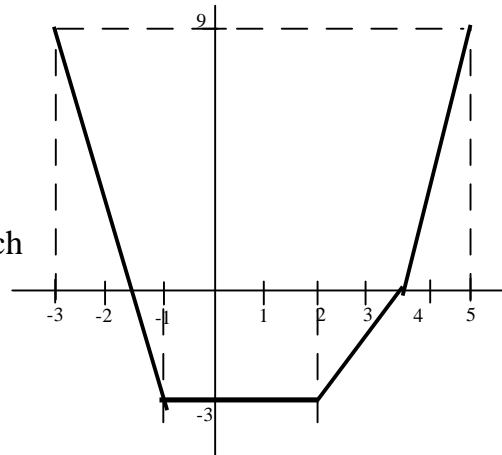
$$2 - x = 0 \Rightarrow x = 2$$

$$2x - 7 = 0 \Rightarrow x = \frac{7}{2}$$

$$1 + x = 0 \Rightarrow x = -1$$

2. Calculate the modulus expressions in each interval

3. Draw the graph



	$\langle -3, -1 \rangle$	$\langle -1, 2 \rangle$	$\langle 2; 3, 5 \rangle$	$\langle 3, 5; 5 \rangle$
$ 2-x $	$2-x$	$2-x$	$x-2$	$x-2$
$ 2x-7 $	$7-2x$	$7-2x$	$7-2x$	$2x-7$
$ 1+x $	$-1-x$	$1+x$	$1+x$	$1+x$
$3 1+x $	$-3-3x$	$3+3x$	$3+3x$	$3+3x$
$ 2-x  +  2x-7  + 3 1+x  - 15$	$-6x-9$	$-3$	$2x-7$	$6x-21$
	$y_1 = -6x-9$ $x \in \langle -3, -1 \rangle$	$y_2 = -3$ $x \in \langle -1, 2 \rangle$	$y_3 = 2x-7$ $x \in \langle 2; 3, 5 \rangle$	$y_4 = 6x-21$ $x \in \langle 3, 5; 5 \rangle$