ELEMENTARY MATHEMATICS is concerned mainly with certain elements called *numbers* and with certain operations defined on them.

NUMERALS
Arabic: 0, 1, 2, 3, 4…
Roman: I, II. III. IV, X, L, C, D, M…

Numerical systems
decadic: 0,1,2,3,4,5,6,7,8,9
binary: 0, 1
ternary: 0, 1, 2

Write the number 1234 in the decimal numerical system: 1*1000 + 2*100 + 3*10 + 4*1
What is the cipher sum of 1287? 1+2+8+7= 18

A constant = invariable: is a symbol that does not change. It is a letter which stands for one number only, e.g. p = 3.14

A variable: is a symbol that changes. It is a letter which stands for a set of numbers, usually it's a letter x, y, y, etc.

Operations
Addition — Sum (the answer in addition), more, more than, increase, increased by, plus
Subtraction — Difference (the answer in subtraction), take away, from, decrease, decreased by, diminish, diminished by, less, less than
Multiplication — Product (the answer in multiplication), double (multiply by 2), triple (multiply by 3), times
Division — Quotient (the answer in division), divided by

Addition	Subtraction	Multiplication	Division
plus the sum of increased by more than added to the total of	minus the difference of decreased by less than fewer than subtracted from	times the product of multiplied by at of	divided by the quotient of the ratio of per

The **( )** symbols are parentheses, the plural of parenthesis. **[ ]** are brackets. **{ }** are braces.

ORDER OF OPERATIONS

Order of Operations

1. Find the values of expressions inside grouping symbols, such as parentheses ( ), brackets [ ], and as indicated by fraction bars.
2. Do all multiplications and/or divisions from left to right.
3. Do all additions and/or subtractions from left to right.

Evaluating expressions

Example 1

$4(1+5)^2 : 8 = 4(6)^2 : 8$ Add 1+5

$= 4(36) : 8$ 6 Squared is 6x6

$= 144 : 8$ Multiply 4 and 36

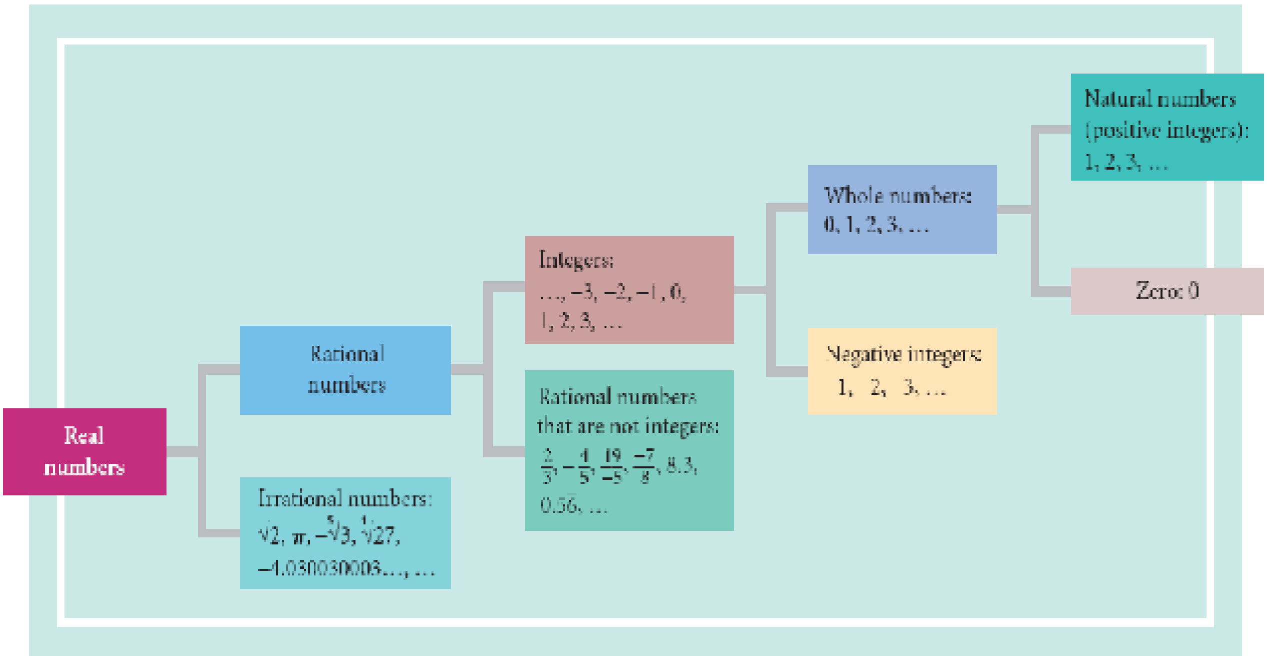
$= 18$ Divide 144 by 8

Evaluate each expression.

1. $3 + 12 + 9$
2. $3 \cdot 5 \cdot 12$
3. $7^2 - 6(2)$
4. $9^2 - 4(9) + 6$
5. $5(4)^2 \div 20 + 9^2$
6. $6(15^2 - 24) - 6(8)$
7. $\frac{13 + 12}{5}$
8. $\frac{5^3 + 19}{16 - 2(4)}$
9. $\frac{3^2 - 2^3}{7 + 2(4)}$

The Number System of Algebra

In applications of algebraic concepts, we use real numbers to represent quantities such as distance, time, speed, area, profit, loss, and temperature. Some frequently used sets of real numbers and the relationships among them are shown below.

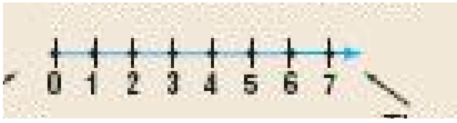


At the beginning, we will deal with two sets of numbers. The first is the set of ***natural numbers*** **N**, which are the numbers 1, 2, 3, 4, . . . and the second is the set of ***whole numbers W*** 0, 1, 2, 3, 4, ... The three dots at the end mean the set is ***infinite***, that it goes on forever. The first four numbers show the pattern.

Numbers like 5.678 or $\frac{3}{4}$ and so on are not natural numbers and not whole numbers.

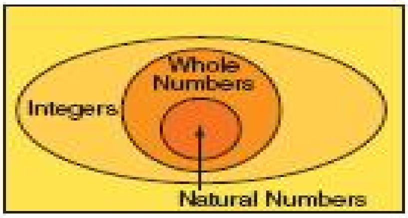
A ***prime*** **natural number** is a natural number with two distinct natural number **factors**, itself and 1.
1 is not a prime. The first eight **prime numbers are 2, 3, 5, 7, 11, 13, 17, and 19**.
9 is not a prime since it has three prime factors: 1, 3, and 9. Numbers like 9 are called ***composites***.

The ***even*** **natural numbers** are the set 2, 4, 6, 8 . . .
The ***odd*** **natural numbers** are the set 1, 3, 5, 7 . . .

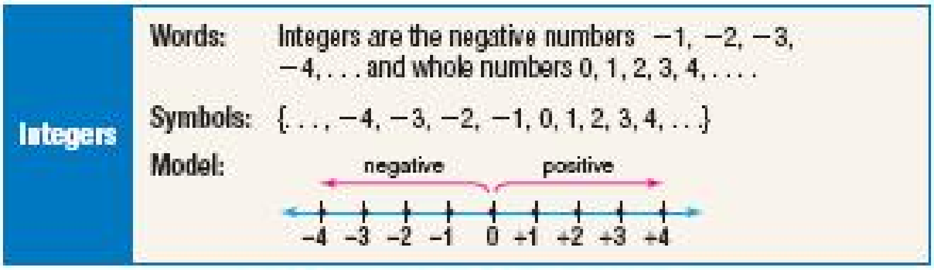


We would like to graph numbers. We will do it on a ***line graph*** or ***number line.***

INTEGERS Z



We prefix each natural number with a + sign to form the **positive integers**, we prefix each natural number with a - sign (the sign must always be written) to form the **negative integers**, and we create a new symbol **0, read zero**.



Zero is neither negative nor positive.

We may state:
Rule 1. To add two numbers having like signs, add their numerical values and prefix their common sign.
Rule 2. To add two numbers having unlike signs, subtract the smaller numerical value from the larger,
and prefix the sign of the number having the larger numerical value.
Rule 3. To subtract a number, change its sign and add.
Rule 4. To multiply or divide two numbers (never divide by O!), multiply or divide the numerical values, prefixing a + sign if the two numbers have like signs and a - sign if the two numbers have unlike signs. (+3)(+2) = +*6*   (-3)(+2) = (+3)(-2) = -6   and   (-3)(-2) = +*6*

THE RATIONAL NUMBERS Q

Rational number can be expressed as a fraction $\frac{p}{q}$, where p **(numerator)** and q **(denominator)** are integers and $b \neq 0$. An **improper fraction** is one where the numerator is larger than the denominator. A **proper fraction** is one where the denominator is larger than the numerator. Two numbers that multiply together to give 1 are **reciprocals** of each other (if we have 2, its reciprocal is $\frac{1}{2}$)

Decimal notation for rational numbers either ***terminates*** (ends) or ***repeats***. Each of the following is a **Rational number.**

Fraction Multiplication
Multiplication of fractions is the easiest of all fraction operations. All you have to do is multiply straight across multiply the numerators (the top numbers) and the denominators (the bottom numbers).

Multiplying Fractions and Whole Numbers
You can multiply fractions by whole numbers in one of two ways:
1. The numerator of the product will be the whole number times the fraction's numerator, and the denominator will be the fraction's denominator.
2. Treat the whole number as a fraction, the whole number over one, and then multiply as you would any two fractions.

Fraction Division
Fraction division is almost as easy as fraction multiplication. Invert (switch the numerator and denominator) the second fraction and the fraction division problem becomes a fraction multiplication problem.

Dividing Fractions		
	Symbols:	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, where $b, c, d \neq 0$
	Numbers:	$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2}$ or $\frac{15}{8}$

Reducing Fractions
When working with fractions, you are usually asked to ''reduce the fraction to lowest terms'' or to ''write the fraction in lowest terms'' or to ''reduce the fraction.'' These phrases mean that the numerator and denominator have no common factors.
Reducing fractions is like fraction multiplication in reverse.

$$\frac{6}{18} = \frac{2 \cdot 3}{2 \cdot 3 \cdot 3} = \frac{(2 \cdot 3) \cdot 1}{(2 \cdot 3) \cdot 3} = \frac{2 \cdot 3}{2 \cdot 3} \cdot \frac{1}{3} = \frac{6}{6} \cdot \frac{1}{3} = 1 \cdot \frac{1}{3} = \frac{1}{3}$$

$$\frac{32}{48} = \frac{16 \cdot 2}{16 \cdot 3} = \frac{16}{16} \cdot \frac{2}{3} = 1 \cdot \frac{2}{3} = \frac{2}{3}$$

Adding and Subtracting Fractions
When adding (or subtracting) fractions with the same denominators, add (or subtract) their numerators.

When the denominators are not the same, you have to rewrite the fractions so that they do have the same denominator. There are two common methods of doing this. The first is the easiest. The second takes more effort but can result in smaller quantities and less reducing. (When the denominators have no common divisors, these two methods are the same.) The easiest way to get a common denominator is to multiply the first fraction by the second denominator over itself and the second fraction by the first denominator over itself.

Compound Fractions

Remember what a fraction is the division of the numerator by the denominator.

$$\frac{\frac{2}{3}}{\frac{1}{6}} = \frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \cdot \frac{6}{1} = \frac{12}{3} = 4$$

$$\frac{1}{\frac{2}{3}} = 1 \div \frac{2}{3} = 1 \cdot \frac{3}{2} = \frac{3}{2}$$

$$\frac{\frac{8}{9}}{5} = \frac{8}{9} \div 5 = \frac{8}{9} \cdot \frac{1}{5} = \frac{8}{45}$$

Mixed Numbers and Improper Fractions

An improper fraction is a fraction whose numerator is larger than its denominator.
To convert a mixed number into an improper fraction, first multiply the whole number by the fraction's denominator. Next add this to the numerator. The sum is the new numerator.

$$2\tfrac{6}{25} = \frac{(2 \cdot 25) + 6}{25} = \frac{50 + 6}{25} = \frac{56}{25}$$

$$1\tfrac{2}{9} = \frac{(1 \cdot 9) + 2}{9} = \frac{11}{9}$$

$$4\tfrac{1}{6} = \frac{(4 \cdot 6) + 1}{6} = \frac{24 + 1}{6} = \frac{25}{6}$$

Comparison Property for Rational Numbers	**Symbols:**	For any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, with $b > 0$ and $d > 0$: 1. If $\frac{a}{b} < \frac{c}{d}$, then $ad < bc$. 2. If $ad < bc$, then $\frac{a}{b} < \frac{c}{d}$.
	Numbers:	1. If $\frac{2}{3} < \frac{3}{4}$, then $8 < 9$. 2. If $8 < 9$, then $\frac{2}{3} < \frac{3}{4}$.

This property also holds true if $<$ is replaced by $>$ or $=$.

DECIMALS

There are two types of decimal numbers, **terminating and nonterminating**. **A nonterminating decimal number** has infinitely many nonzero digits following the decimal point. For example, 0.333333333 . . . is a nonterminating decimal number. Some nonterminating decimal numbers represent fractions—0:333333333 = $\frac{1}{3}$. But some nonterminating decimals, like p =3,1415926654 . . . and $\sqrt{2}$ = 1,414213562 . . ., do not represent fractions.

You can add as many zeros at the end of a terminating decimal number as you want because the extra zeros cancel away.

THE IRRATIONAL NUMBERS

The existence of numbers other than the rational numbers may be inferred from either of the following considerations:

(a) We may conceive of a nonrepeating decimal constructed in endless time by setting down a succession of digits chosen at random.

(b) The length of the diagonal of a square of side 1 is not a rational number, that is, there exists no rational number ***a*** such that $a^2 = 2$. Numbers such as p , e, and ***T*** (but not or ***R)*** are called ***irrational numbers.*** The first three of these are called ***radicals***.

THE REAL NUMBERS

The set of ***real numbers*** consists of the rational and irrational numbers. The real numbers may be ordered by comparing their decimal representations.

We assume that the totality of real numbers may be placed in one-to-one correspondence with the totality of points on a straight line.

THE COMPLEX NUMBERS

Numbers of the form ***a + bi,*** where *a* and *b* are real numbers, are called *complex numbers.* In the complex number *a* + *bi*, ***a* is called the *real part* and *bi* is called the *imaginary part.***

Numbers of the form *ci*, where c is real, are called *imaginary numbers.*

The **complex number *a* + *bi* is a real number when *b* = 0** and **a pure imaginary number when *a* = 0.** When a complex number is not a real number it is called *imaginary.*