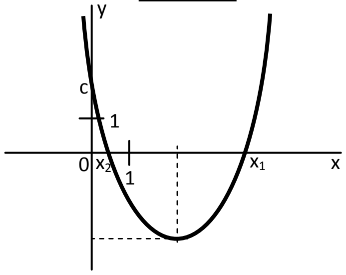
Math Worksheet 7 QUADRATIC FUNCTION

Function equation is; $y = ax^2 + bx + c$**,** the graph of this function is **parable**

a, b, c – coefficients (numbers)

To calculate the intersection with x-axis we calculate the quadratic equation using the following formulae: **Discriminant** $D = b^2 - 4ac$ **Roots**: $x_{1,2} = \dfrac{-b \pm \sqrt{D}}{2a}$ MAX/MIN $\left[-\dfrac{b}{2a}, c - \dfrac{b^2}{4a}\right]$

a > 0

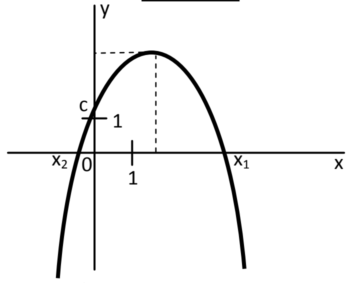


Parable is **convex**, because a>0.

- Domain = R
- Range = $\left\langle c - \dfrac{b^2}{4a}, \infty \right)$
- Decreasing $\left(-\infty, -\dfrac{b}{2a} \right\rangle$

  Increasing $\left\langle -\dfrac{b}{2a}, \infty \right)$
- Bounded below, not bounded above
- In point $x = -\dfrac{b}{2a}$ there is local minimum

a < 0



Parable is **concave**, because a>0.

- Domain = R
- Range = $\left(-\infty, c - \dfrac{b^2}{4a} \right\rangle$
- Increasing $\left(-\infty, -\dfrac{b}{2a} \right\rangle$

  Decreasing $\left\langle -\dfrac{b}{2a}, \infty \right)$
- Bounded above, not bounded below
- In point $x = -\dfrac{b}{2a}$ there is local maximum

EXAMPLE:

Does the curve y = - x² - 4x, have maximum or minimum? Find the roots and sketch the graph.

ROOTS: For finding the roots use the formulae.

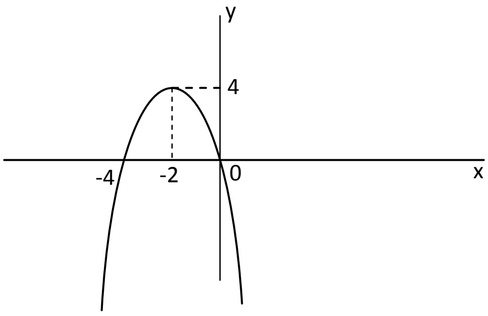
From the equation the coefficients are: **a = -1; b = -4; c = 0**

x₁ = $\dfrac{4 + \sqrt{16 - 4(-1).0}}{2(-1)} = \dfrac{4+4}{(-2)} = -4$ **x₂ =** $\dfrac{-b - \sqrt{b^2 - 4ac}}{2a} = \dfrac{4-4}{(-2)} = 0$

Because we know that a < 0 the curve is concave, so it has the maximum point with the coordinates [x, y] To find this coordinates, we use the formulae: $\left[-\frac{b}{2a}, c-\frac{b^2}{4a}\right]$.

The maximum point has coordinates [x, y]. The value is 4 for the x = -2

MAX [-2, 4]



EXERCISES:

a) $y=2x^2 - 5x + 2$
b) $y = x + 6x + 9$
c) $y = 3x^2 + 7x + 5$
d) $y = 5x^2-25x$
e) $y = 2x^2 + 3x - 2$
f) $y = - x^2 + 2x + 8$
g) $y = 6 - x^2 + 8x$
h) $y = x^2 - 16$
i) $y = 2x^2 + 3x + 1$
j) $y = 3x^2 + 2x + 2$
k) $y = -x^2 + \frac{25}{4}$
l) $y = 5 + 2x - x^2$
m) $y = - 2x^2 - 8x$
n) $y = x^2 + 2$
o) $y = x^2 - 4x + 4$
p) $y = x^2 - 2x$
q) $y = x^2 - x -12$

Challenging exercises *

1. $\frac{2}{2-x} + \frac{x-2}{2} = \frac{x^2}{2(x+2)}$

2. $\frac{x-2}{x-3} = \frac{13}{6} + \frac{3-x}{x-2}$

3. $\frac{1}{x-x^2} = \frac{1-x}{x} - \frac{x}{x-1}$

4. $\frac{3x+3}{3-2x} + \frac{4-x}{2x^2-5x+3} = \frac{2x+1}{1-x}$

5. $\frac{x-2}{x} - \frac{4}{x^2-2x} - \frac{x}{2-x} = 0$

6. $\frac{x-1}{x+1} - \frac{x-2}{x+2} = \frac{x-3}{x+3} - \frac{x-4}{x+4}$

7. $\frac{1}{2x-x^2} + \frac{x-4}{x^2+2x} = \frac{2}{4-x^2}$

SOLUTONS: **1**. 0; **2.** 0;5; **3.** ∅; **4.** 4; **5.** ∅ **6.** 0, $-\frac{5}{2}$; **7.** 3

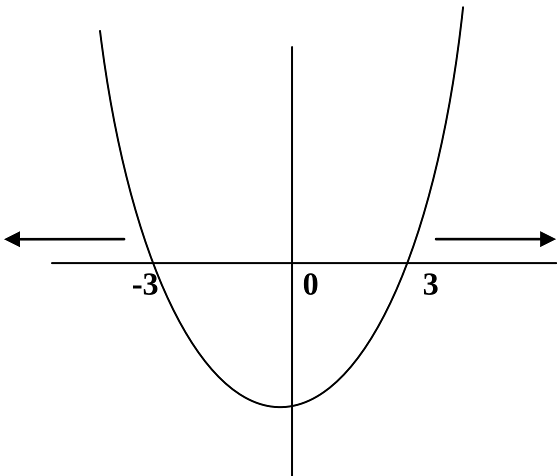
QUADRATIC INEQUALITIES

Let’s explain it using example:

$x^2 - 9 \geq 0$ – we can solve this quadratic inequality either graphically or algebraically

As the first thing calculate the roots of the inequality like with equation using formulae.

Graphically



Arrows indicate that for all values above

3 , 3 included is parabola positive and

for all values below -3 (included) is parabola as well positive . Therefore solution

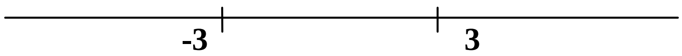
s written below.

(- ∞ , -3 > U < 3 , ∞)

Algebraically

$x^2 - 9 \geq 0$

$(x - 3)(x + 3) \geq 0$



(-).(-) ≥ 0   (-).(+) < 0   (+).(+) ≥ 0

By factorising expression $x^2 - 9$ we get 2 zero values. Numbers 3, -3 divide the number line into 3 intervals. On each of them we do the sign test by substituting values falling into the intervals. It is obvious from the calculation above how it works . Solution is (- ∞ , -3 > U < 3 , ∞)

III. Exercises

1) $x^2 - 4 <$
2) $x^2 - 4x + 3 >$
3) $x^2 - 5x + 6 \leq$
4) $2x^2 + 5x - 3 >$
5) $x^2 - 2x + 1 \leq$
6) $3x^2 + 5x - 2 \leq$
7) $x^2 + 1 \geq 0$
8) $x^2 + 4 < 0$
9) $-x^2 - 6x - 8 \geq 0$
10) $-3x^2 + 4x + 4 > 0$
11) $-2x^2 + 3x \geq 0$