

## Math Worksheet 5 – LINEAR EQUATIONS

For calculating equations and inequalities we use equivalent and non-equivalent algebraic operations:

**Equivalent algebraic operations are:**

- Addition of the number to both sides
- Multiplication of both sides by the same number which doesn't equal 0
- Addition an arbitrary expression

When using only equivalent algebraic operations, the **proof is not needed**.

**Non-equivalent algebraic operations are:**

- Multiplication / division of both sides by an expression
- Exponentiation / Extraction of root

When using non- equivalent algebraic operations, **the proof must be done**.

**For calculating the linear equations use the following formulae:**

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

**Don't forget to set the conditions (domain) where needed.** (Usually if x is in the denominator)

### Exercises

Solve the equations

1.  $(x + 3)(x - 2) + 2(x - 9) = x^2$

2.  $\frac{1}{2}(x - 2) - \frac{2}{3}(2 - x) = \frac{7}{6}$

3.  $\frac{x - 1}{4} - \frac{x - 2}{6} = x$

4.  $[(x + 1) - x]x = x$

5.  $\sqrt{2 + x^2} = x - 1$

6.  $\sqrt{4x - 6} = \sqrt{2x - 4}$

$$7. \frac{5}{x} + \frac{3}{x} + \frac{2}{x} = 20$$

$$8. \frac{1}{x^2} + \frac{1}{2x} = \frac{4}{x}$$

$$9. \frac{x-4}{2(x-1)} + \frac{x+4}{2(x+1)} = 1$$

Solve the following equations in  $\mathbb{Z}_0^+$

$$10. \frac{x+7}{x-11} + \frac{x+11}{x-7} = 2$$

$$11. \frac{x+7}{2x+2} - \frac{x+4}{4x+4} = 1$$

$$12. 3 - \frac{1-2x}{3-x} = \frac{x+3}{x+5}$$

$$13. \frac{x}{2x-3} - \frac{3}{x} = \frac{(x+3)^2}{x(2x-3)}$$

$$14. \frac{2x+3}{4x-5} + \frac{2-x}{5-4x} = 1$$

Solve the following equations in  $\mathbb{R}$

$$15. \frac{2x}{x+3} - \frac{2x}{x-3} = \frac{72}{4x^2-36}$$

$$16. \frac{6}{x+2} + \frac{x+2}{2-x} + \frac{x^2}{x^2-4} = 0$$

$$17. 2-x + \frac{2-x}{3} = \frac{8}{2-x + \frac{2+x}{4}}$$

$$18. \frac{\frac{1}{3} + \frac{1}{x}}{\frac{1}{3} - \frac{1}{x}} = \frac{\frac{x}{3} + 1}{\frac{x}{6}}$$

Solve the following equations in  $\mathbb{N}$

$$19. \frac{17}{x+1} - \frac{5}{x^2+x} = \frac{6}{x}$$

$$20. \frac{x^2+17}{x^2-1} = \frac{x-2}{x+1} - \frac{5}{1-x}$$

### Challenging equations

$$21*. \frac{1}{\sqrt{6+\sqrt{x}}} = \frac{1}{\sqrt{15-2\sqrt{x}}}$$

$$22*. \frac{1}{\sqrt[3]{3x-3}} = \frac{1}{\sqrt[3]{2x+7}}$$

$$23*. 1 + \frac{2x}{x+4} + \frac{27}{2x^2+7x-4} = \frac{6}{2x-1}$$

$$24*. \frac{2x-1}{2x+1} = \frac{2x+1}{2x-1} + \frac{8}{1-4x^2}$$