

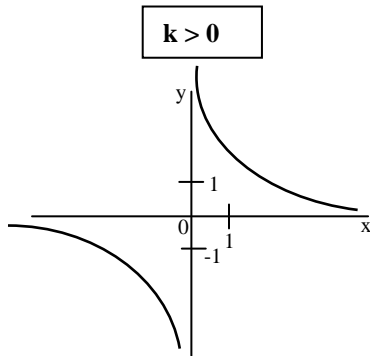
Worksheet 10 RATIONAL FRACTIONAL FUNCTION

$$y = \frac{P(x)}{Q(x)} = \frac{ax+b}{cx+d} = \frac{k}{x}; (k \neq 0)$$

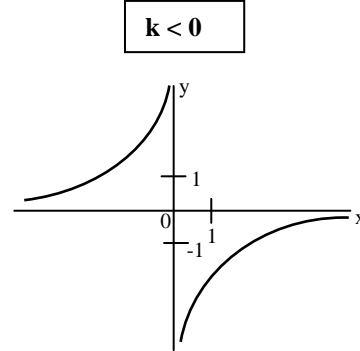
$P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$

$a, b, c, d \in \mathbb{R}; c, d \neq 0$ and $ad \neq bc$; (if $ad = bc$ then it is constant function)

- The graph of this function is **hyperbola**



Decreasing on the intervals
 $(-\infty, 0), (0, \infty)$



Increasing on the intervals
 $(-\infty, 0), (0, \infty)$

Domain = $\mathbb{R} - \{0\}$
Range = $\mathbb{R} - \{0\}$

Not bounded above neither below
No maximum, no minimum
Odd

EXAMPLE: Draw the graph of the function $y = \frac{3-x}{x-1}$ and write all the properties.

- Divide $(3-x)$ by $(x-1)$**

1. For the moment, I'll ignore the other terms and look just at the leading x of the divisor and the leading $-x$ of the dividend.

$$-x+3:(x-1) = -1$$

2. Now I'll take that x , and multiply it through the divisor, $x-1$.

$$-x+3:(x-1) = -1$$

$$\underline{-x+1}$$

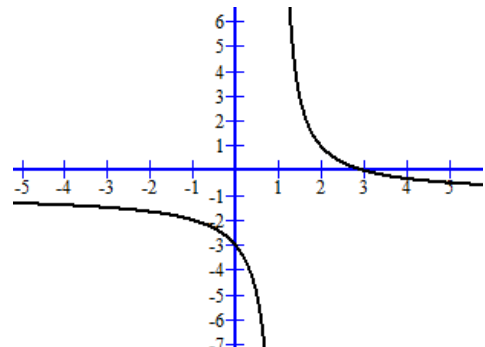
3. To subtract the polynomials, I *change all the signs* in the second line

$$-x+3:(x-1) = -1$$

$$\underline{x-1}$$

$$\text{So I can write } y = \frac{3-x}{x-1} = -1 + \frac{2}{x-1}$$

$D(f) = \mathbb{R} - \{1\}$; $H(f) = \mathbb{R} - \{-1\}$, decreasing, not even, not odd, one-to-one, not bounded, no Min, no Max



EXERCISES: Draw graphs of the following functions and write properties

$$a) y = \frac{2x+3}{x-1},$$

$$b) y = \frac{-x+2}{x-1},$$

$$c) y = \frac{2x-4}{-x+2}$$

$$d) y = \frac{x+3}{x-1},$$

$$e)* y = \frac{5-2x}{3x-1}$$

$$f)* y = \frac{3x+3}{2x-4}$$

Division of polynomials

$$(4x^3 + 5x^2 - 3x + 8) : (2x + 3) = 2x^2 - 0.5x - 0.75 + \frac{8.25}{2x+3}$$

$$\begin{array}{r} \underline{-(4x^3 + 6x^2)} \\ (-x^2 - 3x + 8) \\ \underline{-(-x^2 - 1.5x)} \\ (-1.5x + 8) \\ \underline{-(-1.5x - 2.25)} \\ 8.25 \end{array}$$